

## MAXIMAL NUMERICAL RANGE OF THE BIMULTIPLICATION $M_{2,A,B}$

EL HASSAN BENABDI, MOHAMED KAADOUD CHRAIBI  
AND ABDERRAHIM BAGHDAD\*

**Abstract.** Let  $\mathcal{B}(\mathcal{H})$  denote the  $C^*$ -algebra of all bounded linear operators acting on a complex Hilbert space  $\mathcal{H}$ . For  $A, B \in \mathcal{B}(\mathcal{H})$ , define the bimultiplication operator  $M_{2,A,B}$  on the class of Hilbert-Schmidt operators by  $M_{2,A,B}(X) = AXB$ . It is known [5] that if either  $A$  or  $B$  is hyponormal, then

$$\overline{W(M_{2,A,B})} = \overline{co(W(A)W(B))},$$

where the bar and  $co$  stand for the closure and the convex hull, respectively and  $W(\cdot)$  denotes the numerical range. In this paper, we give some conditions satisfied by  $A$  and  $B$  to have the following equality

$$W_0(M_{2,A,B}) = co(W_0(A)W_0(B)),$$

where  $W_0(\cdot)$  denotes the maximal numerical range.

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