

ON ZEROS OF MATRIX-VALUED ANALYTIC FUNCTIONS

Z. B. MONGA AND W. M. SHAH

Abstract. We extend a result proved by Dirr and Wimmer [IEEE Trans. Automat. Control 52(2007)] for polynomials to the matrix valued analytic functions and thereby obtain generalizations of some well-known results concerning the zero free regions of a class of analytic functions.

Mathematics subject classification (2020): 12D10, 15A18, 30C15.

Keywords and phrases: Matrix polynomial, zero, Eneström-Kakeya theorem, Frobenius norm.

REFERENCES

- [1] L. V. AHLFORS, *Complex analysis: An introduction to the theory of analytic functions of one complex variable*, New York, London 177 (1953).
- [2] G. DIRR AND H. K. WIMMER, *An Eneström-Kakeya theorem for hermitian polynomial matrices*, IEEE Trans. Automat. Control **52**, (2007), 2151–2153.
- [3] G. ENESTRÖM, *Härledning af en allmän formel för antalet pensionärer, som vid en godtycklig tidpunkt förefinnas inom en sluten pensionsklassa*, Öfvers. Vetensk.-Akad. Förh. **50**, (1893), 405–415.
- [4] G. ENESTRÖM, *Remarque sur un théorème relatif aux racines de l'équation $ax_n x^{n-1} + \dots + a_1 x + a_0 = 0$ où tous les coefficients a sont réels et positifs*, Tohoku Math. J. **18**, (1920), 34–36.
- [5] I. GOHBERG, P. LANCASTER AND L. RODMAN, *Matrix Polynomials*, Academic Press, New York, 1982.
- [6] N. K. GOVIL AND Q. I. RAHMAN, *On the Eneström-Kakeya theorem*, Tohoku Math. J. **20**, (2) (1968), 126–136.
- [7] R. A. HORN AND C. R. JOHNSON, *Matrix Analysis*, Cambridge University Press, 2013.
- [8] R. B. GARDNER, N. K. GOVIL, *Eneström-Kakeya theorem and some of its generalizations*, In Current Topics in Pure and Computational Complex Analysis; S. Joshi, M. Dorff, I. Lahiri, Eds.; Springer: New Delhi, India, 2014; pp. 171–200.
- [9] A. JOYAL, G. LABELLE, Q. I. RAHMAN, *On the location of zeros of polynomials*, Cand. Math. Bull. **10**, (1) (1967), 53–63.
- [10] S. KAKEYA, *On the limits of the roots of an algebraic equations with positive coefficients*, Tohoku Math. J. First Ser. **2**, (1912–13), 140–142.
- [11] C. T. LE, T. H. B. DU AND T. D. NGUYEN, *On the location of eigenvalues of matrix polynomials*, Operators and matrices, arXiv preprint arXiv:1703.00747 (2017).
- [12] M. MARDEN, *Geometry of Polynomials*, no. 3. American Mathematical Soc., 1949.
- [13] A. E. TAYLOR AND C. L. DAVID, *Introduction to functional analysis*, vol. 1, New York: Wiley, 1958.