THE WEIGHTED AND THE DAVIS–WIELANDT BEREZIN NUMBER

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Abstract. A functional Hilbert space is the Hilbert space of complex-valued functions on some set $\Theta \subseteq \mathbb{C}$ that the evaluation functionals $\varphi_\lambda(f) = f(\lambda)$, $\lambda \in \Theta$ are continuous on $\mathcal{H}$. The Berezin number of an operator T is defined by $\mathbf{ber}(T) = \sup_{\lambda \in \Theta} |\widetilde{T}(\lambda)| = \sup_{\lambda \in \Theta} |\langle T\hat{k}_\lambda, \hat{k}_\lambda\rangle|$, where the operator T acts on the reproducing kernel Hilbert space $\mathcal{H} = \mathcal{H}(\Theta)$ over some (nonempty) set Θ. In this paper, we defined the weighted Berezin radius and the weighted Berezin norms and then we obtain some related inequalities. It is shown, among other inequalities, that if $T \in \mathscr{L}(\mathcal{H})$ and $t \in [0, 1]$, then

$$\mathbf{ber}_t^2(T) \leqslant (1 - 2t + 2t^2)\|TT^* + T^*T\|_{\mathbf{ber},1} + (1 - 2t)\mathbf{ber}\left(T^2 + T^{*2}\right).$$

Moreover, we generalize the Davis-Wielandt Berezin number and present some inequalities involving this definition.

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