

REVERSE OF FUJII–SEO TYPE LOG–MAJORIZATION AND ITS APPLICATION TO THE TSALLIS RELATIVE ENTROPIES

JIAN SHI*, YING DAI AND JIAHANG XU

Abstract. In this paper, firstly, we shall show reverse of Fujii-SEO type log-majorization, and discuss the equivalence between the log-majorization and Furuta inequality with negative power. At last, we show its application to the Tsallis relative entropies.

Mathematics subject classification (2020): 47A63.

Keywords and phrases: Fujii-SEO type log-majorization, Furuta inequality with negative power, Löwner-Heinz inequality, Tsallis relative entropies.

REFERENCES

- [1] S. ABE, *Monotone decrease of the quantum nonadditive divergence by projective measurements*, Phys. Lett. A **312** (2003), 336–338.
- [2] H. ARAKI, *On an inequality of Lieb and Thirring*, Lett. Math. Phys. **19** (1990), 167–170.
- [3] T. ANDO AND F. HIAI, *Log majorization and complementary Golden-Thompson type inequalities*, Linear Algebra Appl. **197/198** (1994), 113–131.
- [4] N. BEBIANO, R. LEMOS AND J. DA PROVIDÊNCIA, *Inequalities for quantum relative entropy and entropy*, Linear Algebra Appl. **401** (2005), 159–172.
- [5] M. FUJII AND Y. SEO, *Matrix trace inequalities related to the Tsallis relative entropies of real order*, J. Math. Anal. Appl. **498** (2021), 1–15.
- [6] T. FURUTA, *Extension of the Furuta inequality and Ando-Hiai log-majorization*, Linear Algebra Appl. **219** (1995), 139–155.
- [7] E. HEINZ, *Beiträge zur Störungstheorie der Spektralzerlegung*, Math. Ann. **123** (1951), 415–438.
- [8] F. HIAI, *Log-majorization related to Rényi divergences*, Linear Algebra Appl. **563** (2019), 255–276.
- [9] E. KAMEI, *Complements to Furuta inequality, II*, Math. Japan. **45** (1997), 15–23.
- [10] K. LÖWNER, *Über monotone Matrixfunktionen, II*, Math. Z. **38** (1934), 177–216.
- [11] A. MATSUMOTO AND M. FUJII, *Generalizations of reverse Bebianno-Lemos-Providencia inequality*, Linear Algebra Appl. **430** (2009), 1544–1549.
- [12] J. SHI AND D. ZHAO, *Extension of Hiai type log-majorization inequalities*, Math. Inequal. Appl. **23** (2020), 563–568.
- [13] K. TANAHASHI, *The Furuta inequality with negative powers*, Proc. Amer. Math. Soc. **127** (1999), 1683–1692.
- [14] K. YANAGI, K. KURIYAMA AND S. FURUICHI, *Generalized Shannon inequalities based on Tsallis relative operator entropy*, Linear Algebra Appl. **394** (2005), 109–118.
- [15] C. YANG, *An order preserving inequality via Furuta inequality*, Linear Algebra Appl. **331** (2001), 89–100.