A NOTE ON THE A–SPECTRUM OF A–BOUNDED OPERATORS

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Abstract. Let A be a positive bounded operator on a Hilbert space $(\mathcal{H}, \langle ., . \rangle)$ and P be the orthogonal projection on $\mathrm{cl}(\mathcal{R}(A))$. In the present paper, we prove that if A has closed range and $T \in \mathcal{B}_{A^{1/2}}(\mathcal{H})$ then

$$\sigma_A(T) = \sigma(\alpha(T)) \text{ and } \sigma_A(T) \backslash \{0\} = \sigma\left(A^{1/2} T (A^{1/2})^\dagger\right) \backslash \{0\} = \sigma(TP) \backslash \{0\}.$$

In particular, this allows us to prove that $r_A(T) = r(TP) = \sup_{\lambda \in \sigma_A(T)} |\lambda|$, for any A-bounded

operator $T \in \mathcal{B}_{A^{1/2}}(\mathcal{H})$. Moreover, we prove that if $T \in \mathcal{B}_A(\mathcal{H})$ is A-invertible and S is an A-inverse of T, then S belongs also to $\mathcal{B}_A(\mathcal{H})$. Other results are also derived.

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REFERENCES

[1] A. ALAHMARI, M. MABROUK, AND A. ZAMANI, *Further results on the a-numerical range in C*-algebras*, Banach J. Math. Anal. (2022).

[2] M.-L. ARIAS, G. CORACH, AND M. C. GONZALEZ, *Metric properties of projections in semi-Hilbertian spaces*, Integral Equations Operator Theory **62** (2008), no. 1, 11–28.

[3] M.-L. ARIAS, G. CORACH, AND M.-C. GONZALEZ, *Partial isometries in semi-Hilbertian spaces*, Linear Algebra Appl. **428** (2008), no. 7, 1460–1475.

[4] M.-L. ARIAS, G. CORACH, AND M.-C. GONZALEZ, *Lifting properties in operator ranges*, Acta Sci. Math. (Szeged) **75** (2009), 635–65.

[5] A. BAKLOUTI AND M. MABROUK, *Essential numerical ranges of operators in semi-hilbertian spaces*, Ann. Funct. Anal. (2022).

[6] H. BAKLOUTI, K. FEKI, AND O.-A. AHMED, *Joint numerical ranges of operators in semi-Hilbertian spaces*, Linear Algebra Appl. **555** (2018), 266–284.

[7] H. BAKLOUTI AND S. NAMOURI, *Spectral analysis of bounded operators on semi-hilbertian spaces*, Banach J. Math. Anal. (2022).

[8] A. BOURHIM, *On the local spectral properties of weighted shift operators*, Stud. Math. **163** (2003), 41–69.

[9] A. BOURHIM AND M. MABROUK, *a-numerical range on C*-algebras*, Positivity (2021).

[10] L. DE BRANGES AND J. ROVNYAK, *Square summable power series*, Athena series; selected topics in mathematics, Holt, Rinehart and Winston, 1966.

[11] M. MABROUK M. AND A. ZAMANI, *An extension of the a-numerical radius on C*-algebras*, Banach J. Math. Anal. (2023).

[12] G. MURPHY, *C*-algebras and operator theory*, Elsevier Science, 1990.

[13] J.-G. STAMPFLI AND J.-P. WILLIAMS, *Growth conditions and the numerical range in a Banach algebra*, Tohoku Math. J. (2) **20** (1968), no. 4, 417–424.

[14] A. ZAMANI, *A-numerical radius and product of semi-hilbertian operators*, Bull. Iran. Math. Soc. **47** (2021), 371–377.

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