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SPECTRAL REPRESENTATION OF ABSOLUTELY MINIMUM ATTAINING UNBOUNDED NORMAL OPERATORS

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Abstract. Let $T : D(T) \to H_2$ be a densely defined closed operator with domain $D(T) \subset H_1$. We say T to be absolutely minimum attaining if for every non-zero closed subspace M of H_1 with $D(T) \cap M \neq \{0\}$, the restriction operator $T|_M : D(T) \cap M \to H_2$ attains its minimum modulus $m(T|_M)$. That is, there exists $x \in D(T) \cap M$ with $\|x\| = 1$ and $\|T(x)\| = \inf\{\|T(m)\| : m \in D(T) \cap M : \|m\| = 1\}$. In this article, we prove several characterizations of this class of operators and show that every operator in this class has a nontrivial hyperinvariant subspace. One such important characterization is that an unbounded operator belongs to this class if and only if its null space is finite dimensional and its Moore-Penrose inverse is compact.

We also prove a spectral theorem for unbounded normal operators of this class. It turns out that every such operator has a compact resolvent.

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