

PROJECTIONS AND PROPER INFINITENESS FOR CORONA ALGEBRAS

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Abstract. Let \mathcal{B} be a nonunital separable simple Jiang–Su-stable C^* -algebra with stable rank one. We show that $\mathcal{M}(\mathcal{B})$ is the closed linear span of its projections, which implies Property I for \mathcal{B} .

We also show that the corona algebra $\mathcal{C}(\mathcal{B})$ is properly infinite if and only if $T(\mathcal{B})$ is weak* compact. We also provide a number of other equivalent characterizations.

Mathematics subject classification (2020): 47L30, 19K33, 19K35, 19K56.

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REFERENCES

- [1] E. M. ALFSEN, *Compact convex sets and boundary integrals*, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 57, Springer–Verlag, New York–Heidelberg, 1971, doi:10.1007/978-3-642-65009-3.
- [2] E. BLANCHARD, R. ROHDE, M. ROERDAM, *Properly infinite $C(X)$ -algebras and K_1 -injectivity*, Journal of noncommutative geometry **2** (2008), 263–282, doi:doi.org/10.4171/jncg/21.
- [3] L. BROWN, R. DOUGLAS, P. FILLMORE, *Unitary equivalence modulo the compact operators and extensions of C^* -algebras*, Proceedings of a Conference on Operator Theory (Dalhousie Univ., Halifax, N.S., 1973), pp. 58–128, Lecture Notes in Math., vol. 345, Springer, Berlin, 1973.
- [4] L. G. BROWN AND G. K. PEDERSEN, *C^* -algebras of real rank zero*, Journal of Functional Analysis **99** (1991), 131–149.
- [5] N. BROWN, F. PERERA AND A. TOMS, *The Cuntz semigroup, the Elliott conjecture, and dimension functions on C^* -algebras*, J. Reine Angew. Math. **2008** Issue 621, 191–211.
- [6] J. CUNTZ AND G. K. PEDERSEN, *Equivalence and traces on C^* -algebras*, J. Funct. Anal. **33** (1979), no. 2, 135–164.
- [7] K. DAVIDSON, *C^* -algebras by Examples*, The Fields Institute Monographs, 6. American Mathematical Society, Providence, RI, 1996, doi:10.1090/fim/006.
- [8] G. A. ELLIOTT, D. E. HANDELMAN, *Addition of C^* -algebra extensions*, Pacific Journal of Mathematics **137** (1989), no. 1, 87–121. doi:10.2140/pjm.1989.137.87.
- [9] T. FACK, *Finite sums of commutators in C^* -algebras*, Ann. Inst. Fourier (Grenoble) **32** (1982), no. 1, 129–137.
- [10] X. JIANG AND H. SU, *On a simple unital projectionless C^* -algebra*, Amer. J. Math. **121** (1999), no. 2, 359–413.
- [11] G. KASPAROV, *The operator K -functor and extensions of C^* -algebras*, Izv. Akad. Nauk SSSR Ser. Math. **44** (1980), no. 3, 571–636, doi:10.1070/im1981v016n03abeh001320.
- [12] V. KAFTAL, P. W. NG AND S. ZHANG, *Strict comparison of positive elements in multiplier algebras*, Canad. J. Math. **69** (2017), no. 2, 373–407.
- [13] V. KAFTAL, P. W. NG AND S. ZHANG, *The minimal ideal in multiplier algebras*, J. Operator Theory **79** (2018), no. 2, 419–462.
- [14] V. KAFTAL, P. W. NG, S. ZHANG, *Purely infinite corona algebras*, J. Operator Theory **82** (2019), no. 2, 307–355.
- [15] H. LIN, *Simple C^* -algebras with continuous scales and simple corona algebras*, Proceedings of the American Mathematical Society **112** (1991), no. 3, 871–880.

- [16] H. LIN, *Exponential rank of C^* -algebras with real rank zero and the Brown–Pedersen conjectures*, Journal of Functional Analysis **114** (1993), 1–11.
- [17] H. LIN, *Extensions by C^* -algebras of real rank zero, II*, Proc. London Math. Soc. (3) **71** (1995), no. 3, 641–674.
- [18] H. LIN, *Generalized Weyl–von Neumann theorems, II*, Math. Scand. **77** (1995), no. 1, 129–147.
- [19] H. LIN, *Extensions of $C(X)$ by simple C^* -algebras of real rank zero*, Amer. J. Math. **119** (1997), no. 6, 1263–1289.
- [20] H. LIN, *Extensions by C^* -algebras of real rank zero, III*, Proc. London Math. Soc. (3) **76** (1998), no. 3, 634–666.
- [21] H. LIN, *An introduction to the classification of amenable C^* -algebras*, World Scientific Publishing Company, Inc., River Edge, NJ, 2001.
- [22] H. LIN, *Stable approximate unitary equivalence of homomorphisms*, J. Operator Theory, **47** (2002), no. 2, 343–378.
- [23] H. LIN, *Simple corona C^* -algebras*, Proceedings of the American Mathematical Society **132** (2004), no. 11, 3215–3224.
- [24] H. LIN, P. W. NG, *The corona algebra of the stabilized Jiang–Su algebra*, J. Functional Analysis **270** (2016), no. 2, 1220–1267, doi:10.1016/j.jfa.2015.10.020.
- [25] J. LOREAU, P. W. NG, *Remarks on essential codimension*, Integral equations and operator theory **92** (2020), no. 1, Paper no. 4, 35 pp., doi:10.1007/s00020-019-2560-0.
- [26] L. W. MARCOUX, *On the linear span of the projections in certain simple C^* -algebras*, Indiana Univ. Math. J. **51** (2002), no. 3, 753–771.
- [27] P. W. NG, *Nonstable absorption*, Houston Journal of Mathematics, **44** (2018), no. 3, 975–1017.
- [28] P. W. NG, *Remarks on the extension group for purely infinite corona algebras*, Linear and Multilinear Algebra **70** (2022), no. 13, 2459–2504.
- [29] P. W. NG, *Purely infinite corona algebras and extensions*, Journal of Noncommutative Geometry **16** (2022), no. 4, 1363–1395.
- [30] P. W. NG, *Real rank zero for purely infinite corona algebras*, Rocky Mountain Journal of Mathematics **52** (2022), no. 1, 243–261.
- [31] P. W. NG AND L. ROBERT, *Sums of commutators in pure C^* -algebras*, Muenster J. Math. **9** (2016), no. 1, 121–154.
- [32] P. W. NG, T. ROBIN, *Functorial properties of $\text{Ext}(C(X), \mathcal{B})$ when \mathcal{B} is simple with continuous scale*, Journal of Operator Theory, **81** (2019), no. 2, 481–498.
- [33] P. W. NG AND T. ROBIN, *Properly infinite corona algebras*, New York Journal of Mathematics **28** (2022), 69–89.
- [34] L. ROBERT, *On the comparison of positive elements of a C^* -algebra by lower semicontinuous traces*, Indiana Univ. Math. J. **58** (2009), no. 6, 2509–2515.
- [35] L. ROBERT, *Classification of inductive limits of 1-dimensional NCCW complexes*, Adv. Math. **231** (2012) no. 5, 2802–2836, a copy is available at <http://arxiv.org/pdf/1007.1964>.
- [36] K. THOMSEN, *Traces, unitary characters and crossed products by \mathbb{Z}* , Publ. Res. Inst. Math. Sci. **31** (1995), no. 6, 1011–1029.
- [37] A. TOMS AND W. WINTER, *Strongly self-absorbing C^* -algebras*, Trans. Amer. Math. Soc. **359** (2007), no. 8, 3999–4029.
- [38] D. VOICULESCU, *A noncommutative Weyl–von Neumann theorem*, Rev. Roumaine Math. Pures Appl. **21** (1976), 97–113.
- [39] N. E. WEGGE-OLSEN, *K -theory and C^* -algebras. A friendly approach*, Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1993.
- [40] S. ZHANG, *K_1 -groups, quasidiagonality, and interpolation by multiplier projections*, Transactions of the American Mathematical Society **325** (1991), no. 2, 793–818.
- [41] S. ZHANG, *C^* -algebras with real rank zero and their corona and multiplier algebras, IV*, International Journal of Mathematics **3** (1992), no. 2, 309–330.
- [42] S. ZHANG, *Certain C^* -algebras with real rank zero and their corona and multiplier algebras. Part I*, Pacific J. Math. **155** (1992), 169–197.