

SOME INEQUALITIES RELATED TO NUMERICAL RADIUS AND DISTANCE FROM SCALAR OPERATORS IN HILBERT SPACES

MOHAMED CHRAIBI KAADOUD, EL HASSAN BENABDI
AND MESSAOUD GUESBA*

Abstract. In this paper, we characterize bounded linear operators A, B on a complex Hilbert space such that $\inf_{\lambda \in \mathbb{C}} \|A + B - \lambda I\| = \inf_{\lambda \in \mathbb{C}} \|A - \lambda I\| + \inf_{\lambda \in \mathbb{C}} \|B - \lambda I\|$, where I is the identity operator. We also establish some inequalities satisfied by the distance from scalar operators for products of two complex Hilbert space operators.

Mathematics subject classification (2020): 47A05, 47A55, 47B15.

Keywords and phrases: Numerical radius, positive operator, normaloid.

REFERENCES

- [1] A. ABU-OMAR AND F. KITTANEH, *Notes on some spectral radius and numerical radius inequalities*, *Studia Math.* **227**, 2 (2015), 97–109.
- [2] A. ABU-OMAR AND F. KITTANEH, *Numerical radius inequalities for products of Hilbert space operators*, *J. Oper. Theory* **72**, 2 (2014), 521–527.
- [3] T. ANDO, *Distance to the set of thin operators*, unpublished report, 1972.
- [4] M. BARRAA AND M. BOUMAZGOUR, *Inner derivations and norm equality*, *Proc. Amer. Math. Soc.* **130**, (2002), 471–476.
- [5] G. BJÖRCK AND V. THOMÉE, *A property of bounded normal operators in Hilbert space*, *Arkiv Math.* **4**, (1963), 551–555.
- [6] M. L. BUZANO, *Generalizzazione della diseguaglianza di Cauchy-Schwarz*, *Rend. Sem. Mat. Univ. Politech. Torino.*, **31**, 1971/73 (1974), 405–409 (in Italian).
- [7] S. S. DRAGOMIR, *Reverse inequalities for the numerical radius of linear operators in Hilbert spaces*, *Bull. Austral. Math. Soc.* **73**, (2006), 255–262.
- [8] G. GARSKE, *An inequality concerning the smallest disc that contains the spectrum of an operator*, *Proc. Amer. Math. Soc.* **78**, (1980), 529–532.
- [9] M. GUESBA, P. BHUNIA, K. PAUL, *A-numerical radius inequalities and A-translatable radii of semi-Hilbert space operators*, *Filomat* **37**, 11 (2023), 3443–3456.
- [10] K. E. GUSTAFSON AND D. K. M. RAO, *Numerical range*, Universitext, Springer-Verlag, New York, 1997.
- [11] M. S. HOSSEINIA AND B. MOOSAVIB, *Some numerical radius inequalities for products of Hilbert space operators*, *Filomat* **33**, 7 (2019), 2089–2093.
- [12] M. C. KAADOUD, *Géométrie du spectre dans une algèbre de Banach et domaine numérique*, *Studia Math.* **162**, 1 (2004), 1–14.
- [13] M. C. KAADOUD, *Domaine numérique du produit et de la bimultiplication $M_{2,A,B}$* , *Proc. Amer. Math. Soc.* **132**, 8 (2004), 2421–2428.
- [14] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, *Studia Math.* **168**, 1 (2005), 73–80.
- [15] F. KITTANEH, *A numerical radius inequality and an estimate for the numerical radius of the Frobenius companion matrix*, *Studia Math.* **158**, 1 (2003), 11–17.
- [16] J. G. STAMPFLI, *The norm of derivation*, *Pacific J. Math.* **33**, (1970), 737–747.