

SURFACE LOCALIZATION IN IMPURITY BAND WITH ARBITRARY SINGULAR DISORDER AND LONG-RANGE POTENTIALS

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Abstract. We consider a variety of Anderson-type random Hamiltonians in disordered media with a special layer of nonzero co-dimension (“surface” models), and prove spectral and strong dynamical localization in such models, with exponential decay of eigenfunctions and sub-exponential decay of eigenfunction correlators. The main novelty is that we allow arbitrarily singular disorder, and assume that the media-particle interactions feature a power-law decay at infinity.

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