

## SPECTRAL INCLUSION PROPERTY FOR A CLASS OF BLOCK OPERATOR MATRICES

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*Abstract.* The numerical range and the quadratic numerical range is used to study the spectrum of a class of block operator matrices. We show that the approximate point spectrum is contained in the closure of the quadratic numerical range. In particular, the spectral enclosures yield a spectral gap. It is shown that these spectral bounds are tighter than classical numerical range bounds.

*Mathematics subject classification (2020):* 47A10, 35P05, 47A12, 34G10, 47D06, 76B99.

*Keywords and phrases:* Block operator matrices, numerical range, quadratic numerical range, spectrum, spectral enclosures.

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