

CERTAIN PROPERTIES OF T -EP OPERATORS

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Abstract. An operator A is said to be T -EP if $\mathcal{R}(A) = \mathcal{R}(TA^*)$ and $A = AT^*T$, where T is a partial isometry. In this note, some basic properties of T -EP operators are studied. The invariant characterizations that sum and product of two T -EP operators still keep to be T -EP are obtained. As an extension, we obtain necessary and sufficient conditions for a lower triangular operator matrix to be T -EP.

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