Operators and Matrices
Volume 17, Number 4 (2023), 1093–1108
doi:10.7153/oam-2023-17-72

COMPOSITION OPERATORS AND THE CLOSURE OF DIRICHLET–MORREY SPACES IN THE BLOCH SPACE

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Abstract. In this paper, we characterize the closure of the Dirichlet-Morrey spaces in the Bloch space by higher-order derivatives. Moreover, the boundedness and compactness of the products of composition and differentiation operators from the Bloch space to the closure of the Dirichlet-Morrey spaces in the Bloch space are investigated. A criterion for an interpolating Blaschke product to be in the closures is given.

Mathematics subject classification (2020): 47B33, 30H30.
Keywords and phrases: Composition operators, Dirichlet-Morrey spaces, Bloch space, closure.

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Paper OaM-17-72

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