

## GENERALIZED WEIGHTED COMPOSITION OPERATORS ON WEIGHTED HARDY SPACES

LIAN HU, SONGXIAO LI\* AND RONG YANG

*Abstract.* In this paper, we investigate the complex symmetric structure of generalized weighted composition operators  $D_{\psi, \varphi}^m$  on the weighted Hardy space  $H^2(\beta)$ . We obtain explicit conditions for  $D_{\psi, \varphi}^m$  to be complex symmetric with the conjugation  $J_w$ . Under the assumption that  $D_{\psi, \varphi}^m$  is  $J_w$ -symmetric, some sufficient and necessary conditions for  $D_{\psi, \varphi}^m$  to be Hermitian and normal are given.

*Mathematics subject classification (2020):* 30H10, 47B33.

*Keywords and phrases:* Generalized weighted composition operator, weighted Hardy space, complex symmetric, Hermitian, normal.

### REFERENCES

- [1] C. COWEN AND B. MACCLUER, *Composition Operators on Spaces of Analytic Functions*, Studies in Advanced Mathematics, CRC Press, Boca Raton, FL, 1995.
- [2] G. DATT, M. JAIN AND N. OHRI, *On weighted generalized composition operators on weighted Hardy spaces*, *Filomat* **34** (5) (2020), 1689–1700.
- [3] T. EKLUND, M. LINDSTRÖM AND P. MLECZKO, *A note on complex symmetric composition operators on the Bergman space  $A^2(\mathbb{D})$* , *Funct. Approx. Comment. Math.* **59** (1) (2018), 129–139.
- [4] M. FATEHI, *Complex symmetric weighted composition operators*, *Complex Var. Elliptic Equ.* **64** (4) (2019), 710–720.
- [5] M. FATEHI AND C. HAMMOND, *Composition-differentiation operators on the Hardy space*, *Proc. Amer. Math. Soc.* **148** (7) (2020), 2893–2900.
- [6] M. FATEHI AND C. HAMMOND, *Normality and self-adjointness of weighted composition-differentiation operators*, *Complex Anal. Oper. Theory* **15** (1) (2021), 1–13.
- [7] Y. GAO AND Z. ZHOU, *Complex symmetric composition operators induced by linear fractional maps*, *Indiana Univ. Math. J.* **69** (2) (2020), 367–384.
- [8] S. GARCIA AND C. HAMMOND, *Which weighted composition operators are complex symmetric?*, *Concrete operators, spectral theory, operators in harmonic analysis and approximation*, 171–179, *Oper. Theory Adv. Appl.*, 236, Birkhäuser/Springer, Basel.
- [9] S. GARCIA, E. PRODAN AND M. PUTINAR, *Mathematical and physical aspects of complex symmetric operators*, *J. Phys. A* **47** (2014), 533–538.
- [10] S. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications*, *Trans. Amer. Math. Soc.* **358** (3) (2006), 1285–1315.
- [11] S. GARCIA AND M. PUTINAR, *Complex symmetric operators and applications, II*, *Trans. Amer. Math. Soc.* **359** (8) (2007), 3913–3931.
- [12] S. GARCIA AND W. WOGEN, *Complex symmetric partial isometries*, *J. Funct. Anal.* **257** (4) (2009), 1251–1260.
- [13] S. GARCIA AND W. WOGEN, *Some new classes of complex symmetric operators*, *Trans. Amer. Math. Soc.* **362** (11) (2010), 6065–6077.
- [14] A. GUPTA AND A. MALHOTRA, *Complex symmetric weighted composition operators on the space  $H^2(\mathbb{D})$* , *Complex Var. Elliptic Equ.* **65** (9) (2020), 1488–1500.
- [15] K. HAN AND M. WANG, *Weighted composition-differentiation operators on the Hardy space*, *Banach J. Math. Anal.* **15** (3) (2021), 1–18.

- [16] K. HAN AND M. WANG, *Weighted composition-differentiation operators on the Bergman space*, Complex Anal. Oper. Theory **15** (5) (2021), Paper No. 89, 17 pp.
- [17] C. JIANG, S. HAN AND Z. ZHOU, *Complex symmetric weighted composition operators on the Hardy space*, Czechoslovak Math. J. **70** (3) (2020), 817–831.
- [18] S. JUNG, Y. KIM, E. KO AND J. LEE, *Complex symmetric weighted composition operators on  $H^2(\mathbb{D})$* , J. Funct. Anal. **267** (2014), 323–351.
- [19] R. LIM AND L. KHOI, *Complex symmetric weighted composition operators on  $H_\gamma(\mathbb{D})$* , J. Math. Anal. Appl. **464** (1) (2018), 101–118.
- [20] J. LIU, S. PONNUSAMY AND H. XIE, *Complex symmetric weighted composition-differentiation operators*, Linear Mult. Algebra (2022), <https://doi.org/10.1080/03081087.2022.2043816>.
- [21] A. MALHOTRA AND A. GUPTA, *Complex symmetry of generalized weighted composition operators on Fock space*, J. Math. Anal. Appl. **495** (2) (2021), Paper No. 124740, 12 pp.
- [22] A. MALHOTRA AND A. GUPTA, *Complex symmetric weighted composition operators on weighted Hardy space*, Adv. Pure Appl. Math. **13** (1) (2022), 39–49.
- [23] S. NARAYAN, D. SIEVEWRIGHT AND D. THOMPSON, *Complex symmetric composition operators on  $H^2$* , J. Math. Anal. Appl. **443** (1) (2016), 625–630.
- [24] S. NARAYAN, D. SIEVEWRIGHT AND M. TJANI, *Complex symmetric composition operators on weighted Hardy spaces*, Proc. Amer. Math. Soc. **148** (5) (2020), 2117–2127.
- [25] D. THOMPSON, T. MCCLATCHEY AND C. HOLLEMAN, *Binormal, complex symmetric operators*, Linear Mult. Algebra **69** (2021), 1705–1715.
- [26] X. YAO, *Complex symmetric composition operators on a Hilbert space of Dirichlet series*, J. Math. Anal. Appl. **452** (2017), 1413–1419.
- [27] Y. ZHANG, *Essential norm of generalized weighted composition operators from  $H^\infty$  to the logarithmic Bloch space*, J. Integral Equations Appl. **31** (1) (2019), 131–147.
- [28] H. ZHOU AND Z. ZHOU, *Normal complex symmetric weighted composition operators on the Hardy space*, J. Korean Math. Soc. **58** (4) (2021), 799–817.
- [29] X. ZHU, *Products of differentiation, composition and multiplication from Bergman type spaces to Bers type space*, Integ. Tran. Spec. Funct. **18** (2007), 223–231.
- [30] X. ZHU, *Generalized weighted composition operators on weighted Bergman spaces*, Numer. Funct. Anal. Opt. **30** (2009), 881–893.
- [31] X. ZHU, *Generalized weighted composition operators on Bloch-type spaces*, J. Ineq. Appl. **2015** (2015), 59–68.
- [32] X. ZHU, *Essential norm of generalized weighted composition operators on Bloch-type spaces*, Appl. Math. Comput. **274** (2016), 133–142.
- [33] X. ZHU, *Generalized weighted composition operators on weighted Bergman spaces, II*, Math. Inequal. Appl. **22** (3) (2019), 1055–1066.