Operators
and
Matrices
Volume 18, Number 1 (2024), 53–67

doi:10.7153/oam-2024-18-04

THE ALEKSANDROV PROBLEM AND THE TINGLEY PROBLEM FOR EXPANSIVE AND NONEXPANSIVE OPERATORS IN p–NORMED SPACES

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Abstract. In this paper several positive answers are given to the Aleksandrov type problems and the Tingley type problems for some expansive and nonexpansive operators between a real p-normed space and a real q-normed space $(0 < p,q \leqslant 1)$. On the basis of the characteristics of p-normed spaces, the notion of isometry is generalized to the case of with some parameters. It is obtained that some operators of distance preserving can become isometries, and some isometric operators can be extended from the unit sphere to the whole space.

Mathematics subject classification (2020): 46A16, 46A22, 46B04, 51K05.

Keywords and phrases: p-normed space, isometric extension, operator of distance preserving, expansive operator, nonexpansive operator.

REFERENCES

[1] M. AGHAJANI, *On the Mazur-Ulam theorem*, Analysis Math. **44** (2018), 401–407.

[2] F. ALBIAC, *Nonlinear structure of some classical quasi-Banach spaces and F-spaces*, J. Math. Anal. Appl. **340** (2008), 1312–1325.

[3] A. D. ALEKSANDROV, *Mappings of families of sets*, Dokl. Akad. Nauk SSSR **190** (3) (1970), 502–505.

[4] A. BAYOUMI, *Foundations of Complex Analysis in Non Locally Convex Spaces–Function Theory Without Convexity Conditions*, Mathematics Studied **193**, North Holland, Amsterdam, 2003.

[5] L. CHENG AND Y. DONG, *On a generalized Mazur-Ulam question: extension of isometries between unit spheres of Banach spaces*, J. Math. Anal. Appl. **377** (2011), 464–470.

[6] H. Y. CHU, C. G. PARK AND W. G. PARK, *The Alexandrov problem in linear 2-normed spaces*, J. Math. Anal. Appl. **289** (2004), 666–672.

[7] G.-G. DING, *On isometric extension problem between two unit spheres*, Sci. China Ser. A **52** (2009), 2069–2083.

[8] G.-G. DING, *On the linearly isometric extension problem* (in Chinese), Scientia Sinica Mathematica **45** (2015), 1–8.

[9] G.-G. DING AND J.-Z. LI, *Isometric extension problem between strictly convex two-dimensional normed spaces*, Acta Math. Sin. (Engl. Ser.) **35** (2019), 513–518.

[10] X.-N. FANG AND J.-H. WANG, *On extension of isometries between the unit spheres of normed space E and $C(\Omega)$*, Acta Math. Sin. (Engl. Ser.) **22** (2006), 1819–1824.

[11] J.-M. GAO, *On the Alexandrov problem of distance preserving mapping*, J. Math. Anal. Appl. **352** (2009), 583–590.

[12] X.-J. HUANG AND D.-N. TAN, *Mappings of preserving n-distance one in n-normed spaces*, Aequat. Math. **92** (2018), 401–413.

[13] N. J. KALTON, *Quasi-Banach spaces*, Handbook of the Geometry of Banach spaces, Elsevier, Amsterdam, 2003.

[14] H. KHODAEI AND A. MOHAMMADI, *Generalizations of Alesandrov problem and Mazur-Ulam theorem for two-isometries and two-expansive mappings*, Commun. Korean Math. Soc. **34** (2019), 771–782.

[15] C.-W. LEUNG, C.-K. NG AND N.-C. WONG, *On a variant of Tingley's problem for some function spaces*, J. Math. Anal. Appl. **496**: 124800 (2021), 1–16.

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[16] L. LI AND W.-Y. REN, *Isometries on the quasi-Banach spaces L_p $(0 < p < 1)$*, Acta Math. Sin. (Engl. Ser.) **26** (2010), 1519–1524.

[17] Y.-M. MA, *The Aleksandrov problem for unit distance preserving mapping*, Acta. Math. Sci. (Ser. B) **20** (2000), 359–364.

[18] Y.-M. MA, *The Aleksandrov-Benz-Rassias problem on linear n-normed spaces*, Monatsh. Math. **180** (2016), 305–316.

[19] S. MAZUR AND S. ULAM, *Sur les transformations isométriques d'espaces vectoriels normés*, C. R. Acad. Sci. Paris **194** (1932), 946–948.

[20] B. MIELNIK AND T. M. RASSIAS, *On the Aleksandrov problem of conservative distance*, Proc. Amer. Math. Soc. **116** (1992), 1115–1118.

[21] T. M. RASSIAS, *On the A. D. Aleksandrov problem of conservative distances and the Mazur-Ulam theorem*, Nonlinear Anal. **47** (2001), 2597–2608.

[22] T. M. RASSIAS, *Properties of isometric mappings*, J. Math. Anal. Appl. **235** (1999), 108–121.

[23] S. ROLEWICZ, *Metric Linear Spaces*, PWN-Polich Scientific Publishers, Warszawa, 1985.

[24] S. ROLEWICZ, *A generalization of the Mazur-Ulam theorem*, Studia Math. **31** (1968), 501–505.

[25] H. RUI, *On the maps preserving the equality of distance*, J. Math. Anal. Appl. **343** (2008), 1161–1165.

[26] D.-N. TAN, *Nonexpansive mapping and expansive mappings on the unit spheres of some F-spaces*, Bull. Aust. Math. Soc. **82** (2010), 22–30.

[27] D.-N. TAN, *On extension of isometries on the unit spheres of L^p-spaces for $0 < p \leqslant 1$*, Nonlinear Anal. **74** (2011), 6981–6987.

[28] D.-N. TAN, *Phase-isometries on the unit sphere of $C(K)$*, Ann. Funct. Anal. **12**: 15 (2021), 1–14.

[29] D.-N. TAN AND X.-J. HUANG, *Phase-isometries on real normed spaces*, J. Math. Anal. Appl. **488**: 124058 (2020), 1–17.

[30] D. TINGLEY, *Isometries of the unit sphere*, Geom. Dedicata **22** (1987), 371–378.

[31] A. VOGT, *Maps which preserve equality of distance*, Stud. Math. **45** (1973), 43–48.

[32] J. WANG, *On extension of isometries between unit spheres of AL_p-spaces $(0 < p < \infty)$*, Proc. Amer. Math. Soc. **132** (2004), 2899–2909.

[33] S.-M. WANG AND W.-Y. REN, *On the Aleksandrov-Rassias problem of distance preserving mappings*, Bull. Aust. Math. Soc. **91** (2015), 464–470.

[34] J.-Z. XIAO AND Y. LU, *Some fixed point theorems for s-convex subsets in p-normed spaces based on measures of noncompactness*, J. Fixed Point Theory Appl. **20**: 2 (2018), 1–22.

[35] J.-Z. XIAO AND Y. LU, *Extension of isometries between the unit spheres of p-normed spaces*, Ann. Funct. Anal. **13**: 25 (2022), 1–10.

[36] J.-Z. XIAO AND X.-H. ZHU, *Some fixed point theorems for s-convex subsets in p-normed spaces*, Nonlinear Anal. **74** (2011), 1738–1748.

[37] X.-Z. YANG, Z.-B. HOU AND X.-H. FU, *On linear extension of isometries between the unit spheres of β-normed spaces*, Acta Math. Sinica (Chin. Ser.) **48** (2005), 1199–1202.