

## BEREZIN NUMBER INEQUALITIES VIA POSITIVITY OF $2 \times 2$ BLOCK MATRICES

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*Abstract.* Suppose  $\mathcal{B}(\mathcal{H}(\Omega))$  is the set of all bounded linear operators acting on a reproducing kernel Hilbert space  $\mathcal{H}(\Omega)$ . Applying the positivity criteria of  $2 \times 2$  block matrices, we develop several new upper bounds for the Berezin number of operators in  $\mathcal{B}(\mathcal{H}(\Omega))$  involving Berezin norm, which are better than the earlier ones. Among other results, we obtain that if  $T, S \in \mathcal{B}(\mathcal{H}(\Omega))$  and  $0 < \alpha < 1$ , then

$$\mathbf{ber}^p(T \pm S) \leq \frac{1}{2} \left\| \left( |T^*|^{2\alpha} + |S^*|^{2\alpha} \right)^p + \left( |T|^{2(1-\alpha)} + |S|^{2(1-\alpha)} \right)^p \right\|_{ber}, \text{ for all } p \geq 1.$$

By letting  $S = 0$ , it follows that  $\mathbf{ber}^p(T) \leq \frac{1}{2} \left\| |T^*|^{2\alpha p} + |T|^{2(1-\alpha)p} \right\|_{ber}$ , for all  $p \geq 1$ .

*Mathematics subject classification (2020):* 47A30, 15A60, 47A12, 47B65.

*Keywords and phrases:* Berezin number, positive operator, reproducing kernel Hilbert space, operator matrix.

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