

THE INHERITANCE OF m -COMPARISON FROM THE CONTAINING C^* -ALGEBRA TO A LARGE SUBALGEBRA

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Abstract. Let A be a unital simple separable infinite dimensional stably finite C^* -algebra and B be a large subalgebra of A . In this paper, we show that B has (strong tracial or tracial) m -comparison of positive elements if A has (strong tracial or tracial) m -comparison of positive elements.

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