

## FURTHER PROPERTIES OF PPT AND $(\alpha, \beta)$ -NORMAL MATRICES

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**Abstract.** This paper discusses further properties of positive partial transpose matrices, with applications towards normal and  $(\alpha, \beta)$ -normal matrices. The obtained results present extensions and improvements of many results in the literature.

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