

PERTURBATION OF CONTINUOUS FRAMES ON QUATERNIONIC HILBERT SPACES

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Abstract. In this note, we explore the theory of continuous frame perturbations in the quaternion settings by inspiration of the well-developed theory of perturbing discrete frames in complex Hilbert spaces. We examine the perturbation of continuous frames of rank n , including Bessel and Riesz families, within right quaternionic Hilbert spaces. We also investigate some results on the bounds of perturbing continuous frames under some conditions.

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