MULTIPLICATIVELY NUMERICAL RANGE–PRESERVING MAPS

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Abstract. Given a complex Hilbert space $\mathcal{H}$, we denote by $B(\mathcal{H})$ the algebra of all bounded linear operators on $\mathcal{H}$, and $\mathcal{A}$, $\mathcal{B}$ two subsets of $B(\mathcal{H})$ containing all operators of rank at most one. Let $W(A)$ the numerical range of $A \in B(\mathcal{H})$. For an infinite-dimensional space $\mathcal{H}$, we prove that surjective maps $\phi_1, \phi_2 : \mathcal{A} \rightarrow \mathcal{B}$ satisfy

$$W(\phi_1(A)\phi_2(B)) = W(AB), \quad (A, B \in \mathcal{A})$$

if and only if there exist $\mu, \nu \in \mathbb{C}$ with $\mu \nu = 1$, a bounded invertible linear operator $U$ on $\mathcal{H}$ and a unitary operator $V$ on $\mathcal{H}$ such that $\phi_1(A) = \mu V A U^{-1}$ and $\phi_2(A) = \nu U A V^*$ for all $A \in \mathcal{A}$. We also obtain an analogue result for the finite-dimensional case. Furthermore, some known results are obtained as immediate consequences of our main results.


Keywords and phrases: Numerical range, numerical radius, nonlinear preserver.

REFERENCES


