

SOME NOVEL INEQUALITIES FOR BEREZIN NUMBER OF OPERATORS

ULAŞ YAMANCI, MESSAOUD GUESBA AND DUYGU USLU

Abstract. In this paper, some Berezin number inequalities of bounded linear operators defined on a reproducing kernel Hilbert space are developed which generalize and refine the earlier related inequalities. Some applications of the newly obtained inequalities are also provided.

Mathematics subject classification (2020): Primary 47A63.

Keywords and phrases: Berezin number, Berezin symbol, reproducing kernel Hilbert space, inequalities.

REFERENCES

- [1] D. AFRAZ, R. LASHKARIPOUR, M. BAKHERAD, *Further norm and numerical radius inequalities for sum of Hilbert space operators*, Filomat **38** (9) (2024), 3235–3242.
- [2] M. W. ALOMARI, M. HAJMOHAMADI, M. BAKHERAD, *Norm-parallelism of Hilbert space operators and the Davis-Wielandt Berezin number*, J. Math. Inequal. **17** (1) (2023), 231–258.
- [3] M. BAKHERAD, M. T. GARAYEV, *Berezin number inequalities for operators*, Concr Oper. **6** (2019), 33–43.
- [4] M. BAKHERAD, *Some Berezin number inequalities for operator matrices*, Czechoslovak Math. J. **68** (4) (2018), 997–1009.
- [5] M. BAKHERAD M, M. HAJMOHAMADI, R. LASHKARIPOUR, S. SAHOO, *Some extensions of Berezin number inequalities on operators*, Rocky Mountain J. Math. **51** (6) (2021), 1941–1951.
- [6] M. BAKHERAD, F. KITTANEH, *Improved Berezin radius inequalities for certain operator matrices*, Rend. Circ. Mat. Palermo (to appear).
- [7] M. BAKHERAD, F. KITTANEH, *Numerical radius inequalities involving commutators of G_1 operators*, Complex Anal. Oper. Theory **13** (4) (2019), 1557–1567.
- [8] F. A. BEREZIN, *Covariant and contravariant symbols for operators*, Math. USSR-Izv. **6** (1972), 1117–1151.
- [9] F. A. BEREZIN, *Quantizations*, Math. USSR-Izv. **8** (1974), 1109–1163.
- [10] P. BHUNIA, A. SEN, K. PAUL, *Development of the Berezin number inequalities*, Acta. Math. Sin.-English Ser. **39** (2023), 1219–1228.
- [11] P. BHUNIA, K. PAUL, A. SEN, *Inequalities involving Berezin norm and Berezin number*, Complex Anal. Oper. Theory **17** (7) (2023), 1–17.
- [12] M. L. BUZANO, *Generalizzazione della disuguaglianza di Cauchy-Schwarz* (Italian), Rend Sem Mat Univ e Politech Torino. **31** (1974), 405–409.
- [13] S. S. DRAGOMIR, *Inequalities for the norm and the numerical radius of linear operators in Hilbert spaces*, Demonstratio Math. **40** (2) (2007), 411–417.
- [14] S. S. DRAGOMIR, *Inequalities for the numerical radius of linear operators in Hilbert spaces*, Springer Briefs in Mathematics, 2013.
- [15] M. FUJII, R. NAKAMOTO, *Simultaneous extensions of Selberg inequality and Heinz-Kato-Furuta inequality*, Nihonkai Math. J. **9** (2) (1998), 219–225.
- [16] M. T. GARAYEV, M. W. ALOMARI, *Inequalities for the Berezin number of operators and related questions*, Complex Anal. Oper. Theory **15** (2021), 1–30.
- [17] M. GARAYEV, M. BAKHERAD, R. TAPDIGOĞLU, *The weighted and the Davis-Wielandt Berezin number*, Oper. Matrices **17** (2) (2023), 469–484.

- [18] M. HAJMOHAMADI, R. LASHKARIPOUR, M. BAKHERAD, *Improvements of Berezin number inequalities*, Linear Multilinear Algebra **68** (6) (2020), 1218–1229.
- [19] P. R. HALMOS, *A Hilbert Space Problem Book*, 2nd ed., Springer, New York, 1982.
- [20] M. T. KARAEV, *Berezin symbol and invertibility of operators on the functional Hilbert spaces*, J. Funct. Anal. **238** (2006), 181–192.
- [21] M. T. KARAEV, *Reproducing kernels and Berezin symbols techniques in various questions of operator theory*, Complex Anal. Oper. Theory **7** (2013), 983–1018.
- [22] F. KITTANEH, *Notes on some inequalities for Hilbert space operators*, Publ. RIMS Kyoto Univ. **24** (1988), 283–293.
- [23] F. KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math. **168** (1) (2005), 73–80.
- [24] F. KITTANEH, Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **361** (2010), 262–269.
- [25] N. E. MAHDIABADI, M. BAKHERAD, *An extension of the Euclidean Berezin number*, Filomat **37** (24) (2023), 8377–8388.
- [26] S. SAHOO, U. YAMANCI, *Further results on Berezin number inequalities and related problems*, Filomat **37** (30), 10415–10429.
- [27] S. SAHOO, M. BAKHERAD, *Some Extended Berezin Number Inequalities*, Filomat **35** (6) (2021), 2043–2053.
- [28] B. SIMON, *Trace ideals and their applications*, Cambridge University Press, 1979.
- [29] A. TAGHAVI, T. A. ROUSHAN, AND V. DARVISH, *Some upper bounds for the Berezin number of Hilbert space operators*, Filomat **33** (14) (2019), 4353–4360.
- [30] U. YAMANCI, M. GUESBA, *Refinements of some Berezin number inequalities and related questions*, J. Anal. **31** (1) (2023), 539–549.
- [31] U. YAMANCI, M. TAPDIGOĞLU, *Some results related to the Berezin number inequalities*, Turk. J. Math. **43** (4) (2019), 1940–1952.
- [32] U. YAMANCI, M. GARAYEV, C. ÇELİK, *Hardy-Hilbert type inequality in reproducing kernel Hilbert space: its applications and related results*, Linear Multilinear Algebra **67** (2019), 830–842.
- [33] U. YAMANCI, İ. M. KARLI, *Further refinements of the Berezin number inequalities on operators*, Linear Multilinear Algebra **70** (20) (2022), 5237–5246.