

ON LIE TRIPLE CENTRALIZERS OF VON NEUMANN ALGEBRAS

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Abstract. Let \mathcal{U} be a von Neumann algebra endowed with the Lie product $[A, B] = AB - BA$ ($A, B \in \mathcal{U}$). In this article, we consider the subsequent condition on an additive mapping ϕ on the von Neumann algebra \mathcal{U} with a suitable projection $P \in \mathcal{U}$:

$$\phi([A, B], C) = [[\phi(A), B], C] = [[A, \phi(B)], C]$$

for all $A, B, C \in \mathcal{U}$ with $AB = P$ and we show that $\phi(A) = WA + \xi(A)$ for all $A \in \mathcal{U}$, where $W \in Z(\mathcal{U})$, and $\xi: \mathcal{U} \rightarrow Z(\mathcal{U})$ ($Z(\mathcal{U})$ is the center of \mathcal{U}) is an additive map in which $\xi([A, B], C) = 0$ for any $A, B, C \in \mathcal{U}$ with $AB = P$. We also give some results of the conclusion.

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