

SPECTRUM AND FINE SPECTRUM OF BAND MATRICES GENERATED BY OSCILLATORY SEQUENCES

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Abstract. In this paper, a new class of band matrices is considered where the entries of each non-zero band form a sequence with two limit points. The compact perturbation technique is used to study the spectrum over the ℓ_p , ($1 < p < \infty$) sequence space. Several spectral subdivisions such as fine spectrum, discrete spectrum, essential spectrum, etc. are obtained. In addition, a few sufficient conditions on the absence of point spectrum over the essential spectrum are also discussed.

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