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DAVIS–WIELANDT RADIUS INEQUALITIES FOR PRODUCTS OF HILBERT SPACE OPERATORS

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Abstract. In this paper, we obtain some upper bounds for the Davis-Wielandt radius of the product of two bounded linear operators in a Hilbert space, from which we derive new upper bounds for the Davis-Wielandt radius of bounded linear operators. By providing some examples, we show that the upper bounds obtained here are better than the existing upper bounds in some situations.

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