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NUCLEAR VOLTERRA COMPOSITION OPERATORS BETWEEN BANACH TYPE SPACES OF CONTROLLED GROWTH

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Abstract. In this paper, we provide a complete characterization of nuclear Volterra composition operators $T_g^\phi : \mathscr{H}_\nu^\infty \to \mathscr{H}_\mu^\infty$ and $S_g^\phi : \mathscr{H}_\nu^\infty \to \mathscr{H}_\mu^\infty$ acting between weighted type spaces in terms of the symbols g and ϕ, as well as the weights ν and μ, where ν and μ are normal weights in the sense of Shields and Williams. Furthermore, we also characterize nuclear Volterra composition operators acting between little weighted type spaces and Bloch spaces of order β.

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