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INTEGRATION OPERATORS ACTING ON DIFFERENT WEIGHTED BERGMAN SPACES INDUCED BY DOUBLING WEIGHTS

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Abstract. We completely characterize the boundedness of the Volterra-type integration operators T_g acting from the weighted Bergman spaces A^p_ω to A^q_ν for all $0 < p,q < \infty$, $\omega, \nu \in \mathscr{D}$, which extending earlier results by Rättyä. Furthermore, the authors investigates the essential norms of Volterra-type operators between weighted Bergman spaces.

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