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EXTENSION OF THE NOTION OF P–SYMMETRIC OPERATORS USING THE ALUTHGE TRANSFORM II

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Abstract. The class of $\tilde{P}$-symmetric pairs is introduced. Certain properties of this class of operators are obtained. Among other things, it is proved that:

(1) This class includes pairs of quasinormal operators, pairs of co-isometrics operators, pairs of partial isometries with normal squares, and all P-symmetric pairs.

(2) If A and B are two iw-hyponormal operators, then (A,B) has the Fuglede-Putnam property $(FP)_{\mathscr{C}_1(\mathscr{H})}$ if and only if $(\tilde{A},\tilde{B})$ does ($\mathscr{C}_1(\mathscr{H})$ is the ideal of trace class operators).

We also describe some classes of Hilbert space operators A, $B \in \mathscr{L}(\mathscr{H})$ for which we have $\overline{\mathscr{R}\left(\delta_{A,B}\right)}^{w^*} = \overline{\mathscr{R}\left(\delta_{\tilde{A},\tilde{B}}\right)}^{w^*}$, where $\overline{\mathscr{R}\left(\delta_{A,B}\right)}^{w^*}$ is the ultra-weak closure of the range of the generalized derivation $\delta_{A,B}$ defined on $\mathscr{L}(\mathscr{H})$ by $\delta_{A,B}(X) = AX - XB$ and $\tilde{S}$ denote the Aluthge transform of $S \in \mathscr{L}(\mathscr{H})$.

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