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MAPS PRESERVING THE LOCAL SPECTRUM OF GENERALIZED PRODUCT OF OPERATORS

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Abstract. Let $\mathscr{X}$ and $\mathscr{Y}$ be two infinite dimensional Banach spaces and $\mathscr{B}(\mathscr{X})$ (resp. $\mathscr{B}(\mathscr{Y})$) be the algebra of bounded linear operators on $\mathscr{X}$ (resp. $\mathscr{Y}$). For $T \in \mathscr{B}(\mathscr{X})$ and $x \in \mathscr{X}$, $\sigma_T(x)$ denotes the local spectrum of T at x. Fix an integer $k \geqslant 2$, and let $A_1 * A_2 * \ldots * A_k$ stand for a generalized product of any k operators $A_1, A_2, \ldots, A_k \in \mathscr{B}(\mathscr{X})$. Given two nonzero vectors $x_0 \in \mathscr{X}$ and $y_0 \in \mathscr{Y}$, in this paper, we characterize all surjective maps $\varphi : \mathscr{B}(\mathscr{X}) \to \mathscr{B}(\mathscr{Y})$ which satisfy

$$\sigma_{A_1 * A_2 * \ldots * A_k}(x_0) = \sigma_{\varphi(A_1) * \varphi(A_2) * \ldots * \varphi(A_k)}(y_0)$$

for any $A_1, A_2, \ldots, A_k \in \mathscr{B}(\mathscr{X})$.

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