Operators and Matrices
Volume 19, Number 4 (2025), 479–494
doi:10.7153/oam-2025-19-31

SOLVABILITY OF THE SYLVESTER TENSOR EQUATION

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Abstract. Exploring the Sylvester tensor equation not only enhances our understanding of complex systems in various fields but also furnishes us with powerful mathematical instruments to tackle practical challenges. In this study, relying on distinct inequality relationships among the indices that define the positions of elements, we carry out a comprehensive and profound investigation into the solvability of the Sylvester tensor equation by leveraging the theory of block tensors. Furthermore, via rigorous mathematical derivations, we uncover the analytical solution to this arduous equation and offer a thorough analysis of the dimensionality of the solution space.

Mathematics subject classification (2020): 15A18, 15A69.
Keywords and phrases: Sylvester tensor equation, solvability, solution, solution space.

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