Operators and Matrices
Volume 19, Number 4 (2025), 511–532
doi:10.7153/oam-2025-19-33

# UPPER BOUNDS OF NUMERICAL RADIUS AND $a$–NUMERICAL RADIUS IN $\mathcal{C}^*$–ALGEBRA SETTING USING ORLICZ FUNCTIONS

SAIKAT MAHAPATRA, RIDDHICK BIRBONSHI AND ARNAB PATRA

*Abstract.* In this paper, several significant upper bounds for the numerical radius and $a$-numerical radius of an element in a $\mathcal{C}^*$-algebra are obtained using Orlicz functions. Many well-known results are obtained from our findings, depending on specific choices of Orlicz functions.

*Mathematics subject classification* (2020): 46L05, 47A30, 47A12.
*Keywords and phrases*: $\mathcal{C}^*$-algebra, $a$-numerical radius, numerical radius, inequalities, Orlicz function.

## REFERENCES

[1] J. S. AUJLA, SEVER S. DRAGOMIR, M. KHOSRAVI AND M. S, MOSLEHIAN, *Refinements of Choi–Davis–Jensen's inequality*, Bull. Math. Anal. Appl., **3**, 2011.
[2] ANIKET BHANJA, PINTU BHUNIA AND KALLOL PAUL, *On generalized Davis-Wielandt radius inequalities of semi-Hilbertian space operators*, Oper. Matrices, **15** (4): 1201–1225, 2021.
[3] PINTU BHUNIA, *Inequalities for linear functionals and numerical radii on $C^*$-algebras*, Acta Math. Hungar., **176** (1), 111–138, 2025.
[4] PINTU BHUNIA, SILVESTRU SEVER DRAGOMIR, MOHAMMAD SAL MOSLEHIAN AND KALLOL PAUL, *Lectures on numerical radius inequalities*, Springer Nature, 2022.
[5] PINTU BHUNIA, KAIS FEKI AND KALLOL PAUL, *Numerical radius inequalities for products and sums of semi-Hilbertian space operators*, Filomat, **36** (4): 1415–1431, 2022.
[6] PINTU BHUNIA, RAJ KUMAR NAYAK AND KALLOL PAUL, *Improvement of A-numerical radius inequalities of semi-Hilbertian space operators*, Results Math., **76** (3): 120, 2021.
[7] PINTU BHUNIA AND KALLOL PAUL, *Proper improvement of well-known numerical radius inequalities and their applications*, Results Math., **76**: 1–12, 2021.
[8] BRUCE BLACKADAR, *Operator algebras: theory of $C^*$-algebras and von Neumann algebras*, vol. **122**, Springer Science & Business Media, 2006.
[9] ABDELLATIF BOURHIM AND MOHAMED MABROUK, *a-Numerical range on $C^*$-algebras*, Positivity **25**: 1489–1510, 2021.
[10] SEVER S. DRAGOMIR, *Power inequalities for the numerical radius of a product of two operators in Hilbert spaces*, Research report collection, **11** (4), 2008.
[11] SEVER S. DRAGOMIR, *Generalizations of Buzano inequality for n-tuples of vectors in inner product spaces with applications*, Tbilisi Math. J., **10** (2): 29–42, 2017.
[12] SEVER S. DRAGOMIR, *Inequalities for the numerical radius of linear operators in Hilbert spaces*, Springer, 2013.
[13] MOHAMMAD EL-HADDAD AND FUAD KITTANEH, *Numerical radius inequalities for Hilbert space operators, II*, Studia Math., **182**: 133–140, 2007.
[14] KAIS FEKI, *A note on the A-numerical radius of operators in semi-Hilbert spaces*, Archiv der Mathematik, **115** (5): 535–544, 2020.
[15] KAIS FEKI AND FUAD KITTANEH, *Some new refinements of generalized numerical radius inequalities for Hilbert space operators*, Mediterranean Journal of Mathematics, **19** (1): 17, 2022.
[16] TAKAYUKI FURUTA, *Invitation to linear operators: From matrices to bounded linear operators on a Hilbert space*, CRC Press, 2001.


Paper OaM-19-33

[17] FUAD KITTANEH, *Numerical radius inequalities for Hilbert space operators*, Studia Math., **168** (1): 73–80, 2005.
[18] J. LINDENSTRAUSS AND L. TZAFRIRI, *Classical Banach Spaces I and II, Sequence spaces, Function spaces*, Printed in Germany, 1996.
[19] MOHAMED MABROUK AND ALI ZAMANI, *An extension of the a-numerical radius on $C^*$-algebras*, Banach J. Math. Anal., **17** (3): 42, 2023.
[20] AMIT MAJI, ATANU MANNA AND RAM MOHAPATRA, *Orlicz extension of numerical radius inequalities*, arXiv:2207.01915v2[math.FA], 2024.
[21] MOSTAFA SATTARI, MOHAMMAD SAL MOSLEHIAN AND TAKEAKI YAMAZAKI, *Some generalized numerical radius inequalities for Hilbert space operators*, Linear Algebra Appl., **470**: 216–227, 2015.
[22] ALI ZAMANI, *A-numerical radius inequalities for semi-Hilbertian space operators*, Linear Algebra Appl., **578**: 159–183, 2019.
[23] ALI ZAMANI, *Characterization of numerical radius parallelism in $C^*$-algebras*, Positivity, **23** (2): 397–411, 2019.