Operators and Matrices
Volume 20, Number 1 (2026), 1–13
doi:10.7153/oam-2026-20-01

A CHARACTERIZATION OF AN ADDITIVE IDENTITY ON MATRIX ALGEBRAS

ABDUL NADIM KHAN AND MAJED ALBAITY

Abstract. Let $M_n(\mathcal{R})$ be the algebra of $n\times n$ matrices over $\mathcal{R}$, where $\mathcal{R}$ is a commutative two-torsion free ring. In this manuscript, we obtain a structure theorem for the map ψ on $M_n(\mathcal{R})$ satisfying $\psi(\mathscr{A}\mathscr{B}^*\mathscr{A}) = \psi(\mathscr{A})\mathscr{B}^*\mathscr{A} - \mathscr{A}\psi(\mathscr{B})^*\mathscr{A} + \mathscr{A}\mathscr{B}^*\psi(\mathscr{A})$. Moreover, a complete characterization of ψ on $B(\mathcal{H})$, algebra of all bounded linear operators on $\mathcal{H}$, infinite dimensional complex Hilbert space, is given.

Mathematics subject classification (2020): 47B49, 46K15.
Keywords and phrases: $*$-Jordan derivations, matrix algebra.

REFERENCES

[1] M. BREŠAR, *Jordan mappings of semiprime rings*, J. Algebra **127** (1989), 218–228.
[2] L. CHEN AND J. ZHANG, *$*$-Jordan semi-triple derivable mappings*, Indian J. Pure Appl. Math. **53** (2020), 825–837.
[3] W. DU AND J. ZHANG, *Jordan semi-triple derivable maps of matrix algebras*, Acta Math. Sinica **51** (2008), 571–578.
[4] A. FOŠNER AND J. VUKMAN, *On certain functional equations related to Jordan triple (θ,ϕ)-derivations on semiprime rings*, Monatsh Math. **162** (2011), 157–165.
[5] M. FOŠNER AND J. VUKMAN, *On some equations on prime rings*, Monatsh Math. **152** (2007), 135–150.
[6] H. GAO, *$*$-Jordan-triple multiplicative surjective maps on $B(\mathcal{H})$*, J. Math. Anal. Appl. **401** (2013), 397–403.
[7] I. N. HERSTEIN, *Jordan derivations for prime rings*, Proc. Amer. Math. Soc. **8** (1957), 1104–1110.
[8] R. V. KADISON, *Derivations of operator algebras*, Ann. Math. **83** (1966), 280–293.
[9] G. LEŠNJAK AND N. S. SZE, *On injective Jordan semi-triple maps of matrix algebras*, Linear Algebra Appl. **414** (2006), 383–388.
[10] F. LU, *Jordan triple maps*, Linear Algebra Appl. **375** (2003), 311–317.
[11] L. MOLNAR, *On isomorphisms of standard operator algebras*, Stud. Math. **142** (2000), 295–302.
[12] S. SAKAI, *Derivations of W^*-algebras*, **83** (1966), 273–279.
[13] J. VUKMAN, I. KOSI-ULBL AND D. EREMITA, *On certain equations in rings*, Bull. Aus. Math. Soc. **71** (2005), 53–60.

Paper OaM-20-01