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SQUARE ROOTS OF m–COMPLEX SYMMETRIC OPERATORS

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Abstract. A bounded linear operator $T : H \longrightarrow H$ is called m-complex symmetric if there exists a conjugation C on H such that $\Delta_m(T) = 0$, where $\Delta_m(T) = \sum_{j=0}^{m}(-1)^{m-j}\binom{m}{j}T^{*j}CT^{m-j}C$. In this paper, we study the local spectral properties of the square root of an m-complex symmetric operator. First, we show that T^2 is m-complex symmetric if T is m-complex symmetric. Moreover, we study the transfer of some local spectral properties from the Hilbert adjoint T^* to T.

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