

A NEW VIEW ON MULTIPLIERS FOR K -FRAMES IN HILBERT SPACES

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Abstract. In this paper, we give several sufficient conditions under which a K -frame multiplier is invertible and particularly, an equivalent characterization on the invertibility of K -frame multipliers from the perspective of operator theory is presented. We then show that a K -right inverse (resp. K -left inverse) of a K -frame multiplier can be written as a multiplier induced by canonical K -duals, and by means of the corresponding K -duals, we also characterize conditions for a K -right inverse (resp. K -left inverse) to be represented by a multiplier. These results have potential applications in operator sampling theory and signal reconstruction, where K -frames arise naturally.

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