Operators and Matrices
Volume 20, Number 1 (2026), 47–65
doi:10.7153/oam-2026-20-04

SOME INEQUALITIES FOR g–GENERALIZED EUCLIDEAN BEREZIN NUMBER

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Abstract. In this paper, we present several Berezin number inequalities involving upper bounds of the g-generalized Euclidean Berezin number. For example, we show Berezin number inequalities for finite sums of n operators. Among other inequalities, if $T_1,\dots,T_n$, $S_1,\dots,S_n$ are operators in $\mathbb{B}(\mathscr{H}(\Omega))$, then we obtain

$$\mathbf{ber}^r\left(\sum_{i=1}^{n}(T_i+S_i)\right)\leqslant 2^{r-2}\mathbf{ber}(\eta),$$

where $\eta=\sum_{i=1}^{n}\left(f^{2r}|T_i|+f^{2r}|S_i|+g^{2r}|T_i^*|+g^{2r}|S_i^*|\right)$, f and g are nonnegative continuous functions on $[0,\infty)$ such that $f(t)g(t)=t$, $(t\geqslant 0)$, and $r\geqslant 2$. Moreover, we present some results involving hyponormal operators.

Mathematics subject classification (2020): Primary 47A63, 15A18; Secondary 15A45.
Keywords and phrases: Berezin number, g-generalized Berezin number, Hermit-Hadamard inequality, Minkowski's inequality, arithmetic-geometric inequality, convex function.

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