

WEIGHTED NORM ESTIMATES FOR COMMUTATORS OF SEVERAL SQUARE FUNCTIONS

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Abstract. In this article, by using the atomic decomposition of weighted Herz-type Hardy spaces, the authors obtain some strong type estimates for commutators of several square functions generated by BMO functions on these spaces. All these results are also new even for classical Herz-type Hardy spaces.

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