

FURTHER PROPERTIES OF THE MINUS ORDER IN HILBERT SPACES

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Abstract. In this paper, we present the geometric structures of minus order, demonstrating that the minus order is an extension of the star order. The properties of the common minus upper bound are investigated. Some equivalent characterizations of (S, T) -complementability are given, and several properties of the minus shorting operation are obtained.

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