

## ANALYTIC SMOOTHING EFFECT FOR A SYSTEM OF NONLINEAR SCHRÖDINGER EQUATIONS

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*Abstract.* We prove the global existence of analytic solutions to the Cauchy problem for a system of nonlinear Schrödinger equations with quadratic interaction in space dimension  $n \geq 3$  under the mass resonance condition. Lagrangian formulation is also described.

### 1. Introduction

We consider the Cauchy problem for a system of nonlinear Schrödinger equations of the form

$$\begin{cases} i\partial_t u + \frac{1}{2m}\Delta u = \lambda v\bar{u}, \\ i\partial_t v + \frac{1}{2M}\Delta v = \mu u^2, \end{cases} \quad (1.1)$$

where  $u, v$  are complex-valued functions of  $(t, x) \in \mathbb{R} \times \mathbb{R}^n$ ,  $\Delta$  is the Laplacian in  $\mathbb{R}^n$ ,  $m$  and  $M$  are positive constants,  $\lambda, \mu$  are complex constants, and  $\bar{u}$  is the complex conjugate of  $u$ . In this paper, under the mass resonance condition  $M = 2m$ , we study the analyticity of solutions to the Cauchy problem for (1.1) in space dimension  $n \geq 3$  for sufficiently small Cauchy data with exponential decay at infinity. Moreover, we give Lagrangian formulation of the equation (1.1) and characterize the coupling in terms of the gauge structure.

To state our result precisely, we introduce the following notation. For any  $p$  with  $1 \leq p \leq \infty$ ,  $L^p$  denotes the Lebesgue space of  $p$ -th integrable functions on  $\mathbb{R}^n$ . For any  $s \in \mathbb{R}$ ,

$$H_p^s = (1 - \Delta)^{-s/2} L^p \quad \text{and} \quad \dot{H}_p^s = (-\Delta)^{-s/2} L^p$$

denote the usual Sobolev space (or the space of Bessel potentials) and the homogeneous Sobolev space (or the space of Riesz potentials), respectively. For any  $t \in \mathbb{R}$ ,  $U_m(t) = \exp(i\frac{t}{2m}\Delta)$  denotes the free propagator with masses  $m$ . For any  $t \in \mathbb{R}$ ,

$$J_m = J_m(t) = x + i\frac{t}{m}\nabla = U_m(t)xU_m(-t)$$

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denotes the generator of Galilei transforms. For any  $t \in \mathbb{R} \setminus \{0\}$ ,  $U_m(t)$  and  $J_m(t)$  are represented as

$$U_m(t) = M_m(t)D_m(t)\mathcal{F}M_m(t) \quad \text{and} \quad J_m(t) = M_m(t)i\frac{t}{m}\nabla M_m(-t),$$

respectively, where:

$$M_m(t) = \exp(i\frac{m|x|^2}{2t}), \quad (D_m(t)\psi)(x) = (it/m)^{-n/2}\psi(t^{-1}mx),$$

$$(\mathcal{F}\psi)(\xi) = (2\pi)^{-n/2} \int \exp(-ix \cdot \xi)\psi(x)dx.$$

Under the mass resonance condition  $M = 2m$  with the Cauchy data  $(u(0), v(0)) = (\phi, \psi)$  at  $t = 0$ , the Cauchy problem for (1.1) is written as a system of the integral equations

$$\begin{cases} u(t) = U_m(t)\phi - i \int_0^t U_m(t-t')\lambda v\bar{u}(t')dt', \\ v(t) = U_{2m}(t)\psi - i \int_0^t U_{2m}(t-t')\mu u^2(t')dt'. \end{cases} \tag{1.2}$$

For  $n \geq 4$  we introduce the following basic function spaces:

$$\mathcal{X}_0 = L^2(\mathbb{R}; L^{2^*}) \cap L^\infty(\mathbb{R}; L^2),$$

$$\mathcal{X}^\cdot = L^2(\mathbb{R}; \dot{H}_{2^*}^{n/2-2}) \cap L^\infty(\mathbb{R}; \dot{H}_2^{n/2-2}),$$

with the associated norms defined by

$$\|u; \mathcal{X}_0\| = \max(\|u; L_t^2(L^{2^*})\|, \|u; L_t^\infty(L^2)\|),$$

$$\|u; \mathcal{X}^\cdot\| = \max(\|u; L_t^2(\dot{H}_{2^*}^{n/2-2})\|, \|u; L_t^\infty(\dot{H}_2^{n/2-2})\|).$$

where  $2^* = 2n/(n-2)$  is the critical Sobolev exponent. We treat (1.2) in the following function spaces with  $a > 0$ :

$$G_0^a(J_m) = \left\{ u \in \mathcal{X}_0; \|u; G_0^a(J_m)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J_m^\alpha u; \mathcal{X}_0\| < \infty \right\},$$

$$\dot{G}^a(J_m) = \left\{ u \in \mathcal{X}^\cdot; \|u; \dot{G}^a(J_m)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J_m^\alpha u; \mathcal{X}^\cdot\| < \infty \right\},$$

$$G^a(J_m) = G_0^a(J_m) \cap \dot{G}^a(J_m),$$

where  $J_m^\alpha = \prod_{k=1}^n J_{m,k}^{\alpha_k} = M_m(i\frac{t}{m}\partial)^\alpha M_m^{-1}$  for any multi-index  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$ . For  $\rho > 0$ , we define

$$B^a(\rho) = \left\{ (\phi, \psi) \in G^a(x; H_2^{n/2-2}) \times G^{2a}(x; H_2^{n/2-2}); \right.$$

$$\max(\|\phi; G^a(x; \dot{H}_2^{n/2-2})\|, \|\psi; G^{2a}(x; \dot{H}_2^{n/2-2})\|) \leq \rho \Big\},$$

where

$$G^a(x; X) = \left\{ \phi \in X; \|\phi; G^a(x; X)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^\alpha \phi; X\| < \infty \right\}.$$

**THEOREM 1.** *There exists a constant  $\rho > 0$  such that for any  $a > 0$  and  $(\phi, \psi) \in B^a(\rho)$  equation (1.2) has a unique pair of solutions  $(u, v) \in G^a(J_m) \times G^{2a}(J_{2m})$ .*

For  $n = 3$ , we introduce the following basic function space:

$$\mathcal{B}_0 = L^{4,2}(\mathbb{R}, L^3) \cap L^\infty(\mathbb{R}, L^2),$$

with the associated norm defined by

$$\|u; \mathcal{B}_0\| = \max(\|u; L_t^{4,2}(L^3)\|, \|u; L_t^\infty(L^2)\|),$$

where  $L^{4,2}$  is the Lorentz space with second exponent 2, so that  $L^{4,1} \subset L^{4,2} \subset L^{4,4} = L^4 \subset L^{4,\infty}$ . Fractional power of  $|J_m|$  are defined as

$$|J_m|^a(t) = U_m(t)|x|^a U_m(-t), \quad a > 0,$$

which are also represented as (see[8])

$$|J_m|^a(t) = M_m(t) \left( -\frac{t^2}{m^2} \Delta \right)^{a/2} M_m(-t),$$

for  $t \neq 0$ , since  $U_m(t)$  is represented as

$$U_m(t) = M_m(t) D_m(t) \mathcal{F} M_m(t).$$

We define the following Banach space

$$\mathcal{B}_m = \left\{ u \in \mathcal{B}_0; |J_m|^{1/2} u \in \mathcal{B}_0 \right\},$$

with norm

$$\|u; \mathcal{B}_m\| = \max(\|u; \mathcal{B}_0\|, \||J_m|^{1/2} u; \mathcal{B}_0\|).$$

We treat (1.2) in the following function spaces with  $a > 0$  :

$$G_0^a(J_m) = \left\{ u \in \mathcal{B}_0; \|u; G_0^a(J_m)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J_m^\alpha u; \mathcal{B}_0\| < \infty \right\},$$

$$G_m^a(J_m) = \left\{ u \in \mathcal{B}_m; \|u; G_m^a(J_m)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J_m^\alpha u; \mathcal{B}_m\| < \infty \right\},$$

$$G^a(J_m) = G_0^a(J_m) \cap G_m^a(J_m).$$

For  $\rho > 0$ , we define

$$\hat{B}^a(\rho) = \left\{ (\phi, \psi) \in G^a(x; \mathcal{F}(H_2^{1/2})) \times G^{2a}(x; \mathcal{F}(H_2^{1/2})); \right. \\ \left. \max(\|\phi; G^a(x; \mathcal{F}(H_2^{1/2}))\|, \|\psi; G^{2a}(x; \mathcal{F}(H_2^{1/2}))\|) \leq \rho \right\}.$$

**THEOREM 2.** *There exists a constant  $\rho > 0$  such that for any  $a > 0$  and  $(\phi, \psi) \in \hat{B}^a(\rho)$  equation (1.2) has a unique pair of solutions  $(u, v) \in G^a(J_m) \times G^{2a}(J_{2m})$ .*

**REMARK 1.** Theorems 1 and 2 describe analytic smoothing properties of solutions since functions in the space  $G_0^a(J_m)$  are analytic in  $x$  for any  $t \in \mathbb{R} \setminus \{0\}$  (see [11, 12]). The standard scaling argument implies that the Sobolev space  $\dot{H}_2^{n/2-2}$  is critical for (1.1). A novelty consists in the fact that minimal regularity assumption regarding scaling invariance [2, 3, 15, 18] is imposed on the Cauchy data (compare with [12, 17, 27] for instance.)

**REMARK 2.** Mass resonance condition  $M = 2m$  is essential in Theorems 1 and 2 since the proof depends essentially on the identities

$$J_m(v\bar{u}) = 2(J_{2m}v)\bar{u} - v\overline{J_m u}, \\ J_{2m}(u^2) = uJ_m u,$$

which describe the pair of generators of Galilei transforms  $(J_m, J_{2m})$  act on the interactions in (1.1) as if they were derivations.

**REMARK 3.** The parameter  $a$  reminiscent of radius of convergence depends only on  $\rho$  and may be taken uniformly on the data and the corresponding solutions. The ratio  $a : 2a$  for  $u$  and  $v$  comes from the ratio  $m : 2m$  in the mass resonance condition.

**REMARK 4.** Parameter constraint on  $\lambda, \mu$  is unnecessary for the small data setting. Compare with the argument in Section 5.

For a basic literature on nonlinear Schrödinger equations, we refer the reader to [2, 5, 28]. For analytic solutions to Schrödinger equations, we refer the reader to [1, 4, 6, 7, 9, 11, 12, 13, 14, 17, 21, 22, 23, 24, 25, 27, 29].

We prove Theorem 1 in Section 3 by a contraction argument of the Strichartz estimates. We prove Theorem 2 in Section 4 on the basis of the Sobolev embedding in the critical case [26]. Basic estimates for the proofs of the theorems are summarized in Section 2. Lagrangian formulation of the equation is described in Section 5.

## 2. Preliminaries

In this section we collect some basic estimate for the Schrödinger group  $U_m(t) = \exp(i\frac{t}{2m}\Delta)$ , the operator  $\Xi_m$  defined by

$$(\Xi_m f)(t) = \int_0^t U_m(t-t')f(t')dt'$$

and bilinear estimates.

LEMMA 1. ([2, 5, 26, 30].) *Let  $n \geq 4$  then  $U_m(t)$  and  $\Xi_m$  satisfy the following estimates:*

(1) *For any  $(q, r)$  with  $0 \leq 2/q = n/2 - n/r \leq 1$*

$$\|U_m(\cdot)\phi; L^q(\mathbb{R}; L^r)\| \leq C\|\phi; L^2\|.$$

(2) *For any  $(q_j, r_j)$  with  $0 \leq 2/q_j = n/2 - n/r_j \leq 1, j = 1, 2,$*

$$\|\Xi_m f; L^{q_1}(\mathbb{R}; L^{r_1})\| \leq C\|f; L^{q_2}(\mathbb{R}; L^{r_2})\|,$$

where  $p'$  is dual exponent to  $p$  defined by  $1/p + 1/p' = 1$ .

LEMMA 2. ([19, 20].) *Let  $n = 3$  then  $U_m(t)$  and  $\Xi_m$  satisfy the following estimates:*

(1) *For any  $(q, r)$  with  $2 < q < \infty, 2 \leq r \leq \infty, 0 < 2/q = 3/2 - 3/r < 1$*

$$\|U_m(\cdot)\phi; L^{q,2}(\mathbb{R}; L^r)\| \leq C\|\phi; L^2\|.$$

(2) *For any  $(q_j, r_j)$  with  $2 < q_j < \infty, 2 \leq r_j \leq \infty, 0 < 2/q_j = 3/2 - 3/r_j < 1, j = 1, 2,$*

$$\|\Xi_m f; L^{q_j,2}(\mathbb{R}; L^{r_j})\| \leq C\|f; L^{q_j,2}(\mathbb{R}; L^{r_j})\|,$$

where  $p'$  is dual exponent to  $p$  defined by  $1/p + 1/p' = 1$ .

LEMMA 3. ([8].) *Let  $n \geq 4$ . Then there exists a constant  $C$  depending only on  $n$  such that the following estimates hold:*

$$\|uv; L^2\| \leq C\|u; \dot{H}_2^{n/2-2}\| \|v; L^{2*}\|$$

for any  $u \in H_2^{n/2-2}, v \in L^{2*}$  and

$$\|uv; \dot{H}_2^{n/2-2}\| \leq C\|u; \dot{H}_2^{n/2-2}\| \|v; \dot{H}_2^{n/2-2}\|$$

for any  $u, v \in \dot{H}_2^{n/2-2}$

LEMMA 4. ([8].) *Let  $n = 3$ . Then there exists a constant  $C$  depending only on  $n$  such that the following estimates hold:*

$$\begin{aligned} \|uv; L^{3/2}\| &\leq C|t|^{-1/2} \| |J_m|^{1/2}u; L^2 \| \|v; L^3\|, \\ \| |J_m|^{1/2}(v\bar{u}); L^{3/2} \| &\leq C|t|^{-1/2} (\| |J_m|^{1/2}u; L^2 \| \| |J_{2m}|^{1/2}v; L^3 \| \\ &\quad + \| |J_m|^{1/2}u; L^3 \| \| |J_{2m}|^{1/2}v; L^2 \|), \\ \| |J_{2m}|^{1/2}(vu); L^{3/2} \| &\leq C|t|^{-1/2} (\| |J_m|^{1/2}u; L^2 \| \| |J_m|^{1/2}v; L^3 \| \\ &\quad + \| |J_m|^{1/2}u; L^3 \| \| |J_m|^{1/2}v; L^2 \|), \end{aligned}$$

for any  $t \neq 0$  and any  $m > 0$ .

### 3. Proof of Theorem 1.

For  $R, \varepsilon > 0$  we define the metric space

$$\begin{aligned} X(R, \varepsilon) = \left\{ (u, v) \in G^a(J_m) \times G^{2a}(J_{2m}); \max(\|u; G_0^a(J_m)\|, \|v; G_0^{2a}(J_{2m})\|) \leq R, \right. \\ \left. \max(\|u; \dot{G}^a(J_m)\|, \|v; \dot{G}^{2a}(J_{2m})\|) \leq \varepsilon \right\} \end{aligned}$$

with metric

$$d((u, v), (u', v')) = \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|).$$

We see that  $(X(R, \varepsilon), d)$  is a complete metric space. For  $(\phi, \psi) \in B^a(\rho)$  and  $(u, v) \in G^a(J_m) \times G^{2a}(J_{2m})$  we define

$$\begin{aligned} (\Phi(u, v))(t) &= U_m(t)\phi - i(\Xi_m \lambda v \bar{u})(t), \\ (\Psi(u, v))(t) &= U_{2m}(t)\psi - i(\Xi_{2m} \mu u^2)(t). \end{aligned}$$

We prove that the mapping  $(u, v) \mapsto (\Phi(u, v), \Psi(u, v))$  is a contraction on  $(X(R, \varepsilon), d)$  for  $R, \varepsilon > 0$  sufficiently small. By Lemmas 1 and 3, we estimate

$$\begin{aligned} \|\Phi(u, v); \mathcal{X}_0\| &\leq C\|\phi; L^2\| + C\|v\bar{u}; L_t^1(L^2)\| \\ &\leq C\|\phi; L^2\| + C\|v; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|u; L_t^2(L^{2^*})\|, \\ \|\Psi(u, v); \mathcal{X}_0\| &\leq C\|\psi; L^2\| + C\|u; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|u; L_t^2(L^{2^*})\|, \\ \|\Phi(u, v); \mathcal{X}^\cdot\| &\leq C\|\phi; \dot{H}_2^{n/2-2}\| + C\|v; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|u; L_t^2(\dot{H}_{2^*}^{n/2-2})\|, \end{aligned}$$

$$\|\Psi(u, v); \mathcal{X}^\cdot\| \leq C\|\psi; \dot{H}_2^{n/2-2}\| + C\|u; L_t^2(\dot{H}_{2^*}^{n/2-2})\|^2.$$

For any multi-index  $\alpha$ , we have

$$\begin{aligned} J_m^\alpha(\Phi(u, v)) &= U_m(\cdot)x^\alpha\phi - i\lambda \Xi_m(J_m^\alpha(v\bar{u})) \\ &= U_m(\cdot)x^\alpha\phi - i\lambda \Xi_m(M_m(i\frac{t}{m}\partial)^\alpha(M_{2m}^{-1}vM_m\bar{u})) \\ &= U_m(\cdot)x^\alpha\phi - i\lambda \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}(-1)^{|\gamma|}}{\beta!\gamma!} \Xi_m(J_{2m}^\beta v \overline{J_m^\gamma u}), \end{aligned}$$

$$\begin{aligned} J_{2m}^\alpha(\Psi(u, v)) &= U_{2m}(\cdot)x^\alpha\psi - i\mu \Xi_{2m}(J_{2m}^\alpha(u^2)) \\ &= U_{2m}(\cdot)x^\alpha\psi - i\mu \Xi_{2m}(M_{2m}(i\frac{t}{2m}\partial)^\alpha(M_m^{-1}uM_m^{-1}u)) \\ &= U_{2m}(\cdot)x^\alpha\psi - i\mu \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{-|\alpha|}}{\beta!\gamma!} \Xi_{2m}(J_m^\beta u J_m^\gamma u). \end{aligned}$$

By Lemmas 1 and 2, we estimate  $J^\alpha(\Phi(u, v))$  and  $J^\alpha(\Psi(u, v))$  in  $\mathcal{X}_0$  and  $\mathcal{X}^\cdot$  as

$$\begin{aligned} \|J_m^\alpha(\Phi(u, v)); \mathcal{X}_0\| &\leq C\|x^\alpha\phi; L^2\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}}{\beta!\gamma!} \|J_{2m}^\beta v; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|J_m^\gamma u; L_t^2(L^{2^*})\|, \end{aligned}$$

$$\begin{aligned} \|J_{2m}^\alpha(\Psi(u, v)); \mathcal{X}_0\| &\leq C\|x^\alpha\psi; L^2\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{-|\alpha|}}{\beta!\gamma!} \|J_m^\beta u; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|J_m^\gamma u; L_t^2(L^{2^*})\|, \end{aligned}$$

$$\begin{aligned} \|J_m^\alpha(\Phi(u, v)); \mathcal{X}^\cdot\| &\leq C\|x^\alpha\phi; \dot{H}_2^{n/2-2}\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}}{\beta!\gamma!} \|J_{2m}^\beta v; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|J_m^\gamma u; L_t^2(\dot{H}_{2^*}^{n/2-2})\|, \end{aligned}$$

$$\begin{aligned} \|J_{2m}^\alpha(\Psi(u, v)); \mathcal{X}^\cdot\| &\leq C\|x^\alpha\psi; \dot{H}_2^{n/2-2}\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{-|\alpha|}}{\beta!\gamma!} \|J_m^\beta u; L_t^2(\dot{H}_{2^*}^{n/2-2})\| \|J_m^\gamma u; L_t^2(\dot{H}_{2^*}^{n/2-2})\|, \end{aligned}$$

where  $C$  is independent of  $\alpha$ . Multiplying both sides of the above inequalities by  $a^{|\alpha|}/\alpha!$ ,  $(2a)^{|\alpha|}/\alpha!$  and taking the summation over all multi-indices of the resulting inequalities, we have

$$\|\Phi(u, v); G_0^a(J_m)\|$$

$$\begin{aligned}
 &= \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J_m^\alpha(\Phi(u, v)); \mathcal{X}_0\| \\
 &\leq C \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^\alpha \phi; L^2\| + C \sum_{\alpha \geq 0} \sum_{\beta + \gamma = \alpha} \frac{(2a)^{|\beta|} a^{|\gamma|}}{\beta! \gamma!} \|J_{2m}^\beta v; \mathcal{X}\| \|J_m^\gamma u; \mathcal{X}_0\| \\
 &= C \|\phi; G^a(x; L^2)\| + C \|v; \dot{G}^{2a}(J_{2m})\| \|u; G_0^a(J_m)\|,
 \end{aligned}$$

$$\|\Psi(u, v); G_0^{2a}(J_{2m})\| \leq C \|\psi; G^{2a}(x; L^2)\| + C \|u; \dot{G}^a(J_m)\| \|u; G_0^a(J_m)\|,$$

$$\|\Phi(u, v); \dot{G}^a(J_m)\| \leq C \|\phi; G^a(x; \dot{H}_2^{n/2-2})\| + C \|v; \dot{G}^{2a}(J_{2m})\| \|u; \dot{G}^a(J_m)\|,$$

$$\|\Psi(u, v); \dot{G}^a(J_{2m})\| \leq C \|\psi; \dot{G}^{2a}(x; \dot{H}_2^{n/2-2})\| + C \|u; \dot{G}^a(J_m)\|^2.$$

In the same way as above, for  $(u, v), (u', v') \in G^a(J_m) \times G^{2a}(J_{2m})$ , we obtain

$$\begin{aligned}
 &\|\Phi(u, v) - \Phi(u', v'); G^a(J_m)\| \\
 &\leq C(\|u; \dot{G}^a(J_m)\| + \|v'; \dot{G}^{2a}(J_{2m})\|) \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|),
 \end{aligned}$$

$$\begin{aligned}
 &\|\Psi(u, v) - \Psi(u', v'); G^{2a}(J_{2m})\| \\
 &\leq C(\|u; \dot{G}^a(J_m)\| + \|u'; \dot{G}^a(J_m)\|) \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|).
 \end{aligned}$$

Therefore, for any  $(u, v), (u', v') \in X(R, \varepsilon)$  we have

$$\begin{aligned}
 &\max(\|\Phi(u, v); G_0^a(J_m)\|, \|\Psi(u, v); G_0^{2a}(J_{2m})\|) \\
 &\leq C \max(\|\phi; G^a(x; L^2)\|, \|\psi; G^{2a}(x; L^2)\|) + CR\varepsilon,
 \end{aligned}$$

$$\max(\|\Phi(u, v); \dot{G}^a(J_m)\|, \|\Psi(u, v); \dot{G}^{2a}(J_{2m})\|) \leq C\rho + C\varepsilon^2,$$

$$\begin{aligned}
 &\max(\|\Phi(u, v) - \Phi(u', v'); G^a(J_m)\|, \|\Psi(u, v) - \Psi(u', v'); G^{2a}(J_{2m})\|) \\
 &\leq C\varepsilon \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|).
 \end{aligned}$$

and the contraction argument goes through for any  $(\phi, \psi) \in B^a(\rho)$  provided that  $R, \varepsilon$  and  $\rho$  satisfy

$$\begin{cases} C \max(\|\phi; G^a(x; L^2)\|, \|\psi; G^{2a}(x; L^2)\|) + CR\varepsilon \leq R, \\ C\rho + C\varepsilon^2 \leq \varepsilon, \\ C\varepsilon < 1. \end{cases}$$

This completes the proof of Theorem 1.



### 4. Proof of Theorem 2.

For  $R, \varepsilon > 0$  we define the metric space

$$Y(R, \varepsilon) = \left\{ (u, v) \in G^a(J_m) \times G^{2a}(J_{2m}); \max(\|u; G_0^a(J_m)\|, \|v; G_0^{2a}(J_{2m})\|) \leq R, \right. \\ \left. \max(\|u; G_m^a(J_m)\|, \|v; G_{2m}^{2a}(J_{2m})\|) \leq \varepsilon \right\}$$

with metric

$$d((u, v), (u', v')) = \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|).$$

We see that  $(Y(R, \varepsilon), d)$  is a complete metric space. For  $(\phi, \psi) \in \hat{B}^a(\rho)$  and  $(u, v) \in G^a(J_m) \times G^{2a}(J_{2m})$  we define

$$(\Phi(u, v))(t) = U_m(t)\phi - i(\Xi_m \lambda v \bar{u})(t), \\ (\Psi(u, v))(t) = U_{2m}(t)\psi - i(\Xi_{2m} \mu u^2)(t).$$

We prove that the mapping  $(u, v) \mapsto (\Phi(u, v), \Psi(u, v))$  is a contraction on  $(Y(R, \varepsilon), d)$  for  $R, \varepsilon > 0$  sufficiently small. By Lemmas 2 and 4 , we estimate

$$\|\Phi(u, v); \mathcal{B}_0\| \leq C\|\phi; L^2\| + C\|v \bar{u}; L_t^{4/3,2}(L^2)\| \\ \leq C\|\phi; L^2\| + C\|t^{-1/2}\|J_{2m}\|^{1/2}v; L^2\| \|u; L^3\| L_t^{4/3,2} \\ \leq C\|\phi; L^2\| + C\|t^{-1/2}; L_t^{2,\infty}\| \|J_{2m}\|^{1/2}v; L_t^\infty(L^2)\| \|u; L_t^{4,2}(L^3)\| \\ \leq C\|\phi; L^2\| + C\|J_{2m}\|^{1/2}v; L_t^\infty(L^2)\| \|u; L_t^{4,2}(L^3)\|,$$

$$\|\Psi(u, v); \mathcal{B}_0\| \leq C\|\psi; L^2\| + C\|u^2; L_t^{4/3,2}(L^{3/2})\| \\ \leq C\|\psi; L^2\| + C\|J_{2m}\|^{1/2}u; L_t^\infty(L^2)\| \|u; L_t^{4,2}(L^3)\|,$$

$$\|J_m\|^{1/2}\Phi(u, v); \mathcal{B}_0\| \leq C\|x\|^{1/2}\phi; L^2\| + C\|J_m\|^{1/2}(v \bar{u}); L_t^{4/3,2}(L^{3/2})\| \\ \leq C\|\phi; \mathcal{F}(\dot{H}_2^{1/2})\| + C\|J_m\|^{1/2}u; L_t^\infty(L^2)\| \|J_{2m}\|^{1/2}v; L_t^{4,2}(L^3)\| \\ + C\|J_m\|^{1/2}u; L_t^{4,2}(L^3)\| \|J_{2m}\|^{1/2}v; L_t^\infty(L^2)\|,$$

$$\|J_{2m}\|^{1/2}\Psi(u, v); \mathcal{B}_0\| \leq C\|x\|^{1/2}\psi; L^2\| + C\|J_{2m}\|^{1/2}u^2; L_t^{4/3,2}(L^{3/2})\| \\ \leq C\|\psi; \mathcal{F}(\dot{H}_2^{1/2})\| + C\|J_m\|^{1/2}u; L_t^\infty(L^2)\| \|J_m\|^{1/2}u; L_t^{4,2}(L^3)\|.$$

For any multi-index  $\alpha$ , we have as in the proof of Theorem 1

$$J_m^\alpha(\Phi(u, v)) = U_m(\cdot)x^\alpha\phi - i\lambda \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}(-1)^{|\gamma|}}{\beta!\gamma!} \Xi_m(J_{2m}^\beta v \overline{J_m^\gamma u}),$$

$$J_{2m}^\alpha(\Psi(u, v)) = U_{2m}(\cdot)x^\alpha \Psi - i\mu \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{-|\alpha|}}{\beta!\gamma!} \Xi_{2m}(J_m^\beta u J_m^\gamma u).$$

By Lemmas 2 and 4, we estimate

$$J^\alpha(\Phi(u, v)), J^\alpha(\Psi(u, v)), |J_m|^{1/2}J^\alpha(\Phi(u, v)), \quad \text{and} \quad |J_{2m}|^{1/2}J^\alpha(\Psi(u, v)),$$

in  $\mathcal{B}_0$  as

$$\begin{aligned} & \|J_m^\alpha(\Phi(u, v)); \mathcal{B}_0\| \\ & \leq C\|x^\alpha \phi; L^2\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}}{\beta!\gamma!} \| |J_{2m}|^{1/2}J_{2m}^\beta v; L_t^\infty(L^2) \| \| J_m^\gamma u; L_t^{4,2}(L^3) \|, \end{aligned}$$

$$\begin{aligned} & \|J_{2m}^\alpha(\Psi(u, v)); \mathcal{B}_0\| \\ & \leq C\|x^\alpha \Psi; L^2\| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{-|\alpha|}}{\beta!\gamma!} \| |J_m|^{1/2}J_m^\beta u; L_t^\infty(L^2) \| \| J_m^\gamma u; L_t^{4,2}(L^3) \|, \end{aligned}$$

$$\begin{aligned} & \| |J_m|^{1/2}J_m^\alpha(\Phi(u, v)); \mathcal{B}_0 \| \\ & \leq C\| |x|^{1/2}x^\alpha \phi; L^2 \| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}}{\beta!\gamma!} (\| |J_m|^{1/2}J_m^\beta u; L_t^\infty(L^2) \| \| |J_{2m}|^{1/2}J_{2m}^\gamma v; L_t^{4,2}(L^3) \| \\ & \quad + \| |J_m|^{1/2}J_m^\beta u; L_t^{4,2}(L^3) \| \| |J_{2m}|^{1/2}J_{2m}^\gamma v; L_t^\infty(L^2) \|), \end{aligned}$$

$$\begin{aligned} & \| |J_{2m}|^{1/2}J_{2m}^\alpha(\Psi(u, v)); \mathcal{B}_0 \| \\ & \leq C\| |x|^{1/2}x^\alpha \Psi; L^2 \| + C \sum_{\beta+\gamma=\alpha} \frac{\alpha!2^{|\beta|}}{\beta!\gamma!} \| |J_m|^{1/2}J_m^\beta u; L_t^\infty(L^2) \| \| |J_m|^{1/2}J_m^\gamma u; L_t^{4,2}(L^3) \|, \end{aligned}$$

where  $C$  is independent of  $\alpha$ . Multiplying both sides of the above inequalities by  $a^{|\alpha|}/\alpha!$ ,  $(2a)^{|\alpha|}/\alpha!$  and taking the summation over all multi-indices of the resulting inequalities, we have

$$\begin{aligned} & \| \Phi(u, v); G_0^a(J_m) \| \\ & = \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \| J_m^\alpha(\Phi(u, v)); \mathcal{B}_0 \| \\ & \leq C \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \| x^\alpha \phi; L^2 \| + C \sum_{\alpha \geq 0} \sum_{\beta+\gamma=\alpha} \frac{(2a)^{|\beta|}a^{|\gamma|}}{\beta!\gamma!} \| J_{2m}^\beta v; \mathcal{B}_{2m} \| \| J_m^\gamma u; \mathcal{B}_0 \| \\ & = C\|\phi; G^a(x; L^2)\| + C\|v; G_{2m}^{2a}(J_{2m})\| \|u; G_0^a(J_m)\|, \end{aligned}$$

$$\|\Psi(u, v); G_0^{2a}(J_{2m})\| \leq C\|\psi; G^{2a}(x; L^2)\| + C\|u; G_m^a(J_m)\| \|u; G_0^a(J_m)\|,$$

$$\begin{aligned} \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \| |J_m|^{1/2} J_m^\alpha(\Phi(u, v)); \mathcal{B}_0 \| \\ \leq C\|\phi; G^a(x; \mathcal{F}(\dot{H}_2^{1/2}))\| + C\|v; G_{2m}^{2a}(J_{2m})\| \|u; G_m^a(J_m)\|, \end{aligned}$$

$$\begin{aligned} \sum_{\alpha \geq 0} \frac{(2a)^{|\alpha|}}{\alpha!} \| |J_{2m}|^{1/2} J_{2m}^\alpha(\Psi(u, v)); \mathcal{B}_0 \| \\ \leq C\|\psi; G^{2a}(x; \mathcal{F}(\dot{H}_2^{1/2}))\| + C\|u; G_m^a(J_m)\|^2. \end{aligned}$$

In the same way as above, for  $(u, v), (u', v') \in G^a(J_m) \times G^{2a}(J_{2m})$ , we obtain

$$\begin{aligned} \|\Phi(u, v) - \Phi(u', v'); G^a(J_m)\| \\ \leq C(\|u; G_m^a(J_m)\| + \|v'; G_{2m}^{2a}(J_{2m})\|) \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|), \end{aligned}$$

$$\begin{aligned} \|\Psi(u, v) - \Psi(u', v'); G^{2a}(J_{2m})\| \\ \leq C(\|u; G_m^a(J_m)\| + \|u'; G_m^a(J_m)\|) \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|). \end{aligned}$$

Therefore, for any  $(u, v), (u', v') \in Y(R, \varepsilon)$  we have:

$$\begin{aligned} \max(\|\Phi(u, v); G_0^a(J_m)\|, \|\Psi(u, v); G_0^{2a}(J_{2m})\|) \\ \leq C \max(\|\phi; G^a(x; L^2)\|, \|\psi; G^{2a}(x; L^2)\|) + CR\varepsilon, \end{aligned}$$

$$\max(\|\Phi(u, v); G_m^a(J_m)\|, \|\Psi(u, v); G_{2m}^{2a}(J_{2m})\|) \leq C\rho + C\varepsilon^2,$$

$$\begin{aligned} \max(\|\Phi(u, v) - \Phi(u', v'); G^a(J_m)\|, \|\Psi(u, v) - \Psi(u', v'); G^{2a}(J_{2m})\|) \\ \leq C\varepsilon \max(\|u - u'; G^a(J_m)\|, \|v - v'; G^{2a}(J_{2m})\|). \end{aligned}$$

and the contraction argument goes through for any  $(\phi, \psi) \in \hat{B}^a(\rho)$  provided that  $R, \varepsilon$  and  $\rho$  satisfy

$$\begin{cases} C \max(\|\phi; G^a(x; L^2)\|, \|\psi; G^{2a}(x; L^2)\|) + CR\varepsilon \leq R, \\ C\rho + C\varepsilon^2 \leq \varepsilon, \\ C\varepsilon < 1. \end{cases}$$

This completes the proof of Theorem 2.

### 5. Lagrangian Formalism

In this section, we study (1.1) in Lagrangian formalism. We rewrite (1.1) as

$$\begin{cases} i\ddot{u} + \frac{1}{2m}\Delta u = \lambda v\bar{u}, \\ i\ddot{v} + \frac{1}{2M}\Delta v = \mu u^2, \end{cases} \tag{5.1}$$

where  $\dot{u} = \partial_t u$  and  $\dot{v} = \partial_t v$ .

We regard  $(u, \bar{u}, v, \bar{v})$  as well as  $(\dot{u}, \bar{\dot{u}}, \dot{v}, \bar{\dot{v}})$  and  $(\nabla u, \overline{\nabla u}, \nabla v, \overline{\nabla v})$  as independent variables. In [10] the following identities are proved:

$$\begin{aligned} & \|u(t); L^2\|^2 + c\|v(t); L^2\|^2 \\ &= \|u(0); L^2\|^2 + c\|v(0); L^2\|^2 + 2\text{Im} \int_0^t (\lambda - c\bar{\mu})(v(t'), u^2(t')) dt', \\ & \frac{1}{2m} \|\nabla u(t); L^2\|^2 + \frac{c}{4M} \|\nabla v(t); L^2\|^2 \\ &= \frac{1}{2m} \|\nabla u(0); L^2\|^2 + \frac{c}{4M} \|\nabla v(0); L^2\|^2 \\ & \quad - \text{Re} \int_0^t (\lambda(v(t'), \partial_t(u)^2(t') + c\bar{\mu}(\partial_t v(t'), u^2(t'))) dt' \end{aligned}$$

for any  $t, c \in \mathbb{R}$ , where  $(\cdot, \cdot)$  is the scalar product in  $L^2$ . This implies that the charge and energy defined respectively by

$$\begin{aligned} Q(t) &= \|u(t); L^2\|^2 + c\|v(t); L^2\|^2, \\ E(t) &= \frac{1}{2m} \|\nabla u(t); L^2\|^2 + \frac{c}{4M} \|\nabla v(t); L^2\|^2 + \text{Re}(\lambda(v(t), u^2(t))) \end{aligned}$$

are conserved if and only if there exists  $c \in \mathbb{R}$  such that  $\lambda = c\bar{\mu}$ . It is therefore natural to assume the condition  $\lambda = c\bar{\mu}$  with  $c \in \mathbb{R} \setminus \{0\}$  in the variational setting. Under the last assumption, we introduce the following Lagrangian density

$$\mathcal{L} = \frac{i}{c}(\bar{u}\dot{u} - u\dot{\bar{u}}) + \frac{i}{2}(\bar{v}\dot{v} - v\dot{\bar{v}}) - \frac{1}{mc}\nabla u \cdot \overline{\nabla u} - \frac{1}{2M}\nabla v \cdot \overline{\nabla v} - V, \tag{5.2}$$

where

$$V(u, \bar{u}, v, \bar{v}) = \mu u^2 \bar{v} + \bar{\mu} \bar{u}^2 v. \tag{5.3}$$

Then (5.1) is derived as the Euler-Lagrange equation for  $\mathcal{L}$ . We should remark that the mass resonance condition is irrelevant to the variational structure for (5.1).

We give a characterization of the nonlinear potential (5.3) by the gauge structure.

**THEOREM 3.** *Let  $V : \mathbb{C} \times \mathbb{C} \times \mathbb{C} \times \mathbb{C} \ni (u, \bar{u}, v, \bar{v}) \mapsto V(u, \bar{u}, v, \bar{v}) \in \mathbb{R}$  be a cubic homogeneous polynomial with complex coefficients. Then the following conditions are equivalent:*

- (1) *There exists  $\mu \in \mathbb{C}$  such that  $V(u, \bar{u}, v, \bar{v}) = \mu u^2 \bar{v} + \bar{\mu} u^2 v$ .*
- (2)  *$V(u, \bar{u}, v, \bar{v}) = V(e^{i\theta} u, \overline{e^{i\theta} u}, e^{2i\theta} v, \overline{e^{2i\theta} v})$  for all  $\theta \in \mathbb{R}$ ,  $u, v \in \mathbb{C}$ .*

*Proof.* We prove that (2) implies (1), since the converse follows immediately. Since  $V$  is real valued and cubic in  $(u, \bar{u}, v, \bar{v})$ ,  $V$  is written as

$$V(u, \bar{u}, v, \bar{v}) = \sum_{|\alpha|=3} (C_\alpha u^{\alpha_1} \bar{u}^{\alpha_2} v^{\alpha_3} \bar{v}^{\alpha_4} + \overline{C_\alpha} \bar{u}^{\alpha_1} u^{\alpha_2} \bar{v}^{\alpha_3} v^{\alpha_4}),$$

where  $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in \mathbb{Z}_{\geq 0}^4$  is a multi-index of the length 3 and  $C_\alpha \in \mathbb{C}$ . Then the gauge condition (2) yields the following simultaneous linear equations of  $\alpha$  :

$$\begin{cases} \alpha_1 - \alpha_2 + 2\alpha_3 - 2\alpha_4 = 0, \\ \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 3. \end{cases}$$

We see that the solutions are  $(2, 0, 0, 1)$  and  $(0, 2, 1, 0)$  only. This implies (1) with  $\mu = C_{(2,0,0,1)} + \overline{C_{(0,2,1,0)}}$ .  $\square$

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