Differential
Equations
& Applications
Volume 6, Number 2 (2014), 267–274
doi:10.7153/dea-06-13

ON POSITIVE SOLUTION FOR A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS WITH SIGN–CHANGING WEIGHTS

DIANPENG HE AND ZUODONG YANG

(*Communicated by Dongsheng Kang*)

Abstract. In this paper, we consider the problem for the existence of positive solutions of quasilinear elliptic system

$$\begin{cases} -\Delta_p u = \lambda a(x) u^{\alpha} v^{\gamma}, & x \in \Omega, \\ -\Delta_q v = \lambda b(x) u^{\eta} v^{\beta}, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases}$$

where the $\lambda > 0$ is a parameter, Ω is a bounded domain in $\mathbb{R}^N (N > 1)$ with smooth boundary $\partial\Omega$, and the $\Delta_p z = \operatorname{div}(|\nabla z|^{p-2}\nabla z)$ is the p-Laplacian operator. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe are negative near the boundary. Using the method of sub-super solutions and comparison principle, which studied the existence of positive solutions for quasilinear elliptic system. The main results of the present paper are new and extend the previously known results.

1. Introduction

In this note we consider the existence of positive solutions for the system

$$\begin{cases} -\Delta_p u = \lambda a(x) u^{\alpha} v^{\gamma}, & x \in \Omega, \\ -\Delta_q v = \lambda b(x) u^{\eta} v^{\beta}, & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\lambda > 0$ is a parameter, $1 < p,q < N$, and Ω is a bounded domain in $\mathbb{R}^N (N > 1)$ with smooth boundary $\partial\Omega$, and the $\Delta_p z = \operatorname{div}(|\nabla z|^{p-2}\nabla z)$ is the p-Laplacian operator. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe are negative near the boundary.

Problem (1.1) arises in the theory of quasi-regular and quasi-conformal mappings as well as in the study of non- Newtonian fluids. In the latter case, the pair (p,q) is a characteristic of the medium. Media with $(p,q) > (2,2)$ are called dilatant fluids and those with $(p,q) < (2,2)$ are called pseudo-plastics. If $(p,q) = (2,2)$, they are Newtonian fluids.

Mathematics subject classification (2010): 35J65, 35J50.
Keywords and phrases: quasilinear elliptic systems, sign-changing weight, positive weak solution.
This research is supported by the National Natural Science Foundation of China(No.11171092); the Natural Science Foundation of the Jiangsu Higher Education Institutions of China(No.08KJB110005).

When $p=q=2$, the following system

$$\begin{cases} \Delta u = a(|x|)v^{\alpha}, & x \in \mathbb{R}^N, \\ \Delta v = b(|x|)u^{\beta}, & x \in \mathbb{R}^N, \end{cases}$$

for which existence results for boundary blow-up positive solutions can be found in a recent paper by Lair and Wood [12]. The authors established that all positive entire radial solutions of systems above are boundary blow-up provided that

$$\int_0^{\infty} ta(t)dt = \infty, \qquad \int_0^{\infty} tb(t)dt = \infty.$$

On the other hand, if

$$\int_0^{\infty} ta(t)dt < \infty, \qquad \int_0^{\infty} tb(t)dt < \infty,$$

then all positive entire radial solutions of this system are bounded.

F. Cìrstea and V.Rădulescu [5] extended the above results to a larger class of systems

$$\begin{cases} \Delta u = a(|x|)g(v), & x \in \mathbb{R}^N, \\ \Delta v = b(|x|)f(u), & x \in \mathbb{R}^N. \end{cases}$$

Z.D.Yang [15] extended the above results to a class of systems

$$\begin{cases} \mathrm{div}(|\nabla u|^{p-2}\nabla u) = a(|x|)g(v), & x \in \mathbb{R}^N, \\ \mathrm{div}(|\nabla v|^{q-2}\nabla v) = b(|x|)f(u), & x \in \mathbb{R}^N. \end{cases}$$

Caisheng Chen [3] discussed the existence and non-existence of positive weak solution to the following system

$$\begin{cases} -\Delta u = \lambda u^{\alpha} v^{\gamma}, & x \in \Omega, \\ -\Delta v = \lambda u^{\delta} v^{\beta}, & x \in \Omega, \\ u(x) = v(x) = 0, & x \in \partial\Omega. \end{cases}$$

D.D. Hai [11] studied the existence and nonexistence of positive solutions for the quasilinear system

$$\begin{cases} -\Delta_p u = \lambda a(x) f(u,v), & x \in \Omega, \\ -\Delta_q v = \mu b(x) g(u,v), & x \in \Omega, \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{E}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega, p, q > 1, \lambda, \mu$ are positive parameters, $a(x), b(x)$ are bounded functions that can change sign, which obtained existence results for the quasilinear system (E) when $f(t,t)$ is p-sublinear in 0 and $g(t,t)$ is q-sublinear at 0, and λ, μ are small. Nonexistence results are also obtained.

Motivated by the above results, we focus on further extending the study in [3] to the system (1.1) and supplementary the results in [11]. In fact, we study the existence of positive solution to the system (1.1) with sign-changing weight functions $a(x)$ and $b(x)$. Due to this weight functions, the existence are challenging and nontrivial. Our approach is based on the method of sub-super solutions, see [3, 14].

To precisely state our existence result we need the eigenvalue problem

$$-\Delta_p\phi_1 = \lambda_1|\phi_1|^{p-2}\phi_1,\ \ x\in\Omega,\ \ \phi_1 = 0,\ \ x\in\partial\Omega, \tag{1.2}$$

$$-\Delta_q\phi_1^* = \lambda_1^*|\phi_1^*|^{q-2}\phi_1^*,\ \ x\in\Omega,\ \ \phi_1^* = 0,\ \ x\in\partial\Omega. \tag{1.3}$$

Let $\lambda_1 > 0$ be the principal eigenvalue and $\phi_1 > 0$ with $\|\phi_1\|_\infty = 1$ the corresponding eigenfunction of $-\Delta_p$ and $\lambda_1^* > 0$ be the principal eigenvalue and $\phi_1^* > 0$ with $\|\phi_1^*\|_\infty = 1$ the corresponding eigenfunction of $-\Delta_q$, with the Dirichlet boundary condition. It is well known that

$$\frac{\partial\phi_1}{\partial\nu} < 0 \quad \text{and} \quad \frac{\partial\phi_1^*}{\partial\nu} < 0 \quad \text{on } \partial\Omega,$$

where ν is the unit outward normal, while $\phi_1, \phi_1^* = 0$ on $\partial\Omega$. This result is well known and hence, depending on Ω, there exist $\sigma, \sigma^* \in (0,1], \delta > 0$ and $m > 0$ such that (see [14])

$$\lambda_1\phi_1^p - |\nabla\phi_1|^p \leqslant -m,\ \ \text{on } \overline{\Omega}_\delta, \tag{1.4}$$

$$\phi_1 \geqslant \sigma,\ \ \text{on } \Omega_0 = \Omega\backslash\overline{\Omega}_\delta, \tag{1.5}$$

and

$$\lambda_1\phi_1^{*q} - |\nabla\phi_1^*|^q \leqslant -m,\ \ \text{on } \overline{\Omega}_\delta, \tag{1.6}$$

$$\phi_1^* \geqslant \sigma^*,\ \ \text{on } \Omega_0 = \Omega\backslash\overline{\Omega}_\delta, \tag{1.7}$$

where $\Omega_\delta = \{x\in\Omega | d(x,\partial\Omega) < \delta\}$. We will also consider the unique solution, $e_1(x)$, $e_2(x) \in C^1(\overline{\Omega})$, of the boundary value problem

$$-\Delta_p e_1 = 1,\ \ x\in\Omega,\ \ e_1 = 0,\ \ x\in\partial\Omega, \tag{1.8}$$

$$-\Delta_q e_2 = 1,\ \ x\in\Omega,\ \ e_2 = 0,\ \ x\in\partial\Omega, \tag{1.9}$$

to discuss our existence result. It is known that $e_i(x) > 0 (i = 1,2)$ in Ω and

$$\frac{\partial e_i(x)}{\partial\nu} < 0 \quad \text{on } \partial\Omega\,(i = 1,2) \quad \text{(see [8, 9, 10])}.$$

2. Existence results

In this section, we shall establish our existence result via the method of sub and supersolutions. A pair of nonnegative functions (ψ_1, ψ_2), (z_1, z_2), are called a subsolution and supersolution of (1.1) if they satisfy $(\psi_1, \psi_2) = (0,0) = (z_1, z_2)$, on $\partial\Omega$

$$\int_\Omega |\nabla\psi_1|^{p-2}|\nabla\psi_1|\cdot\nabla f_1 dx \leqslant \lambda\int_\Omega a(x){\psi_1}^\alpha{\psi_2}^\gamma f_1 dx,$$

$$\int_\Omega |\nabla\psi_2|^{q-2}|\nabla\psi_2|\cdot\nabla f_2 dx \leqslant \lambda\int_\Omega b(x){\psi_1}^\eta{\psi_2}^\beta f_2 dx,$$

and

$$\int_\Omega |\nabla z_1|^{p-2}|\nabla z_1| \cdot \nabla f_1 dx \geqslant \lambda \int_\Omega a(x) {z_1}^\alpha {z_2}^\gamma f_1 dx,$$

$$\int_\Omega |\nabla z_2|^{p-2}|\nabla z_2| \cdot \nabla f_2 dx \geqslant \lambda \int_\Omega a(x) {z_1}^\eta {z_2}^\beta f_2 dx,$$

for all test functions $f_1(x) \in W_0^{1,p}(\Omega)$ and $f_2(x) \in W_0^{1,q}(\Omega)$ with $f_1, f_2 \geqslant 0$. Then the following result holds:

LEMMA 1. *(See* [17]*) Suppose there exist sub and super-solutions* (ψ_1, ψ_2) *and* (z_1, z_2) *respectively of (1.1) such that* $(\psi_1, \psi_2) \leqslant (z_1, z_2)$. *Then (1.1) has a solution* (u, v) *such that* $(u, v) \in [(\psi_1, \psi_2), (z_1, z_2)]$.

We make the following assumptions:

(i) $\alpha, \beta \geqslant 0$, $\gamma, \eta > 0$ and $(p-1-\alpha)(q-1-\beta) > \gamma\eta$;

(ii) Assume that there exist positive constants a_0, a_1, b_0 and b_1, such that

$$a(x) \geqslant -a_0, b(x) \geqslant -b_0 \ \text{ on } \overline{\Omega}_\delta$$

and

$$a(x) \geqslant a_1, b(x) \geqslant b_1 \ \text{ on } \Omega \backslash \overline{\Omega}_\delta;$$

(iii) Suppose that there exists $\varepsilon > 0$ such that:

$$\frac{\lambda_1}{m} a_0 < \min\left\{c_1, c_2 \varepsilon^{d_2 - d_1}\right\}, \quad \frac{\lambda_1}{m} a_0 < \min\left\{c_1 \varepsilon^{-(d_2 - d_1)}, c_2\right\}$$

and

$$\max\left\{\frac{\lambda_1}{c_1}\varepsilon^{1-d_1}, \frac{\lambda_1^*}{c_2}\varepsilon^{1-d_2}\right\} \leqslant \min\left\{\frac{1}{\|a\|_\infty}, \frac{1}{\|b\|_\infty}\right\},$$

where

$$c_1 = a_1 \left(\frac{p-1}{p}\sigma^{\frac{p}{p-1}}\right)^\alpha \left(\frac{q-1}{q}\sigma^{*\frac{q}{q-1}}\right)^\gamma, \ c_2 = b_1 \left(\frac{p-1}{p}\sigma^{\frac{p}{p-1}}\right)^\eta \left(\frac{q-1}{q}\sigma^{*\frac{q}{q-1}}\right)^\beta,$$

$$d_1 = \frac{\alpha}{p-1} + \frac{\gamma}{q-1}, \quad d_2 = \frac{\eta}{p-1} + \frac{\beta}{q-1}.$$

Now we are ready to state our existence results.

THEOREM 1. *Let* (i) – (iii) *hold. Then there exists a positive solution of (1.1) for every* $\lambda \in [\underline{\lambda}(\varepsilon), \overline{\lambda}(\varepsilon)]$, *where*

$$\overline{\lambda} = \min\left\{\frac{m}{a_0\varepsilon^{d_1-1}}, \frac{m}{b_0\varepsilon^{d_1-1}}, \frac{1}{\|a\|_\infty}, \frac{1}{\|b\|_\infty}\right\}, \tag{2.1}$$

$$\underline{\lambda} = \max\left\{\frac{\lambda_1}{c_1\varepsilon^{d_1-1}}, \frac{\lambda_1^*}{c_2\varepsilon^{d_2-1}}\right\}. \tag{2.2}$$

REMARK 1. Note that (iii) implies $\underline{\lambda} < \overline{\lambda}$.

Proof. Let

$$(\psi_1, \psi_2) = (\frac{p-1}{p}\varepsilon^{\frac{1}{p-1}}\phi_1^{\frac{p}{p-1}}, \frac{q-1}{q}\varepsilon^{\frac{1}{q-1}}\phi_1^{*\frac{q}{q-1}}),$$

we shall verify that (ψ_1, ψ_2) is a sub-solution of (1.1). Let $f_1 \in W_0^{1,p}(\Omega)$, then a calculation shows that

$$\begin{aligned}\int_\Omega |\nabla\psi_1|^{p-2}\nabla\psi_1\cdot\nabla f_1 dx &= \varepsilon\int_\Omega \phi_1|\nabla\phi_1|^{p-2}\nabla\phi_1\cdot\nabla f_1 dx\\ &= \varepsilon\left\{\int_\Omega |\nabla\phi_1|^{p-2}\nabla(\phi_1 f_1)dx - \int_\Omega |\nabla\phi_1|^p f_1 dx\right\}\\ &= \varepsilon\left\{\int_\Omega [\lambda_1\phi_1^p - |\nabla\phi_1|^p] f_1 dx\right\}.\end{aligned} \tag{2.3}$$

A similarly calculation shows that

$$\int_\Omega |\nabla\psi_2|^{q-2}\nabla\psi_2\cdot\nabla f_2 dx = \varepsilon\left\{\int_\Omega [\lambda_1^*\phi_1^{*q} - |\nabla\phi_1^*|^q] f_2 dx\right\}. \tag{2.4}$$

First, we consider the case when $x \in \overline{\Omega}_\delta$. We have $\lambda_1\phi_1^p - |\nabla\phi_1|^p \leqslant -m$ on $\overline{\Omega}_\delta$ and since $\lambda \leqslant \overline{\lambda}$, we have $\lambda \leqslant \frac{m}{a_0\varepsilon^{d_1-1}}$. Then

$$-\varepsilon m \leqslant -\lambda a_0 \varepsilon^{\frac{\alpha}{p-1}+\frac{\gamma}{q-1}}. \tag{2.5}$$

Hence

$$\begin{aligned}\varepsilon(\lambda_1\phi_1^p - |\nabla\phi_1|^p) &\leqslant -m\varepsilon\\ &\leqslant -\lambda a_0\varepsilon^{\frac{\alpha}{p-1}+\frac{\gamma}{q-1}}\\ &\leqslant -\lambda a_0(\frac{p-1}{p}\varepsilon^{\frac{1}{p-1}}\|\phi_1\|_\infty^{\frac{p}{p-1}})^\alpha(\frac{q-1}{q}\varepsilon^{\frac{1}{q-1}}\|\phi_1^*\|_\infty^{\frac{q}{q-1}})^\gamma\\ &\leqslant \lambda a(x)\psi_1^\alpha\psi_2^\gamma.\end{aligned} \tag{2.6}$$

A similar argument shows that:

$$\varepsilon(\lambda_1^*\phi_1^{*q} - |\nabla\phi_1^*|^q) \leqslant \lambda b(x)\psi_1^\eta\psi_2^\beta. \tag{2.7}$$

Then we obtain from (2.3) , (2.4) and (2.6) , (2.7) that

$$\int_{\overline{\Omega}_\delta} |\nabla\psi_1|^{p-2}|\nabla\psi_1|\cdot\nabla f_1 dx \leqslant \lambda\int_{\overline{\Omega}_\delta} a(x){\psi_1}^{\alpha}{\psi_2}^{\gamma} f_1 dx, \tag{2.8}$$

$$\int_{\overline{\Omega}_\delta} |\nabla\psi_2|^{q-2}|\nabla\psi_2|\cdot\nabla f_2 dx \leqslant \lambda\int_{\overline{\Omega}_\delta} b(x){\psi_1}^{\eta}{\psi_2}^{\beta} f_2 dx. \tag{2.9}$$

On the other hand, on $\Omega\backslash\overline{\Omega}_\delta$, we note that

$$\phi_1 \geqslant \sigma > 0,\ \phi_1^* \geqslant \sigma^* > 0,\ a(x) \geqslant a_1,\ b(x) \geqslant b_1$$

and since $\lambda \geqslant \underline{\lambda}$, we have $\lambda \geqslant \frac{\lambda_1}{c_1\varepsilon^{d_1-1}}$. Then

$$\varepsilon\lambda_1 \leqslant \lambda a_1\Big(\frac{p-1}{p}\varepsilon^{\frac{1}{p-1}}\sigma^{\frac{p}{p-1}}\Big)^{\alpha}\Big(\frac{q-1}{q}\varepsilon^{\frac{1}{q-1}}\sigma^{*\frac{q}{q-1}}\Big)^{\gamma}. \tag{2.10}$$

Hence

$$\begin{aligned}\varepsilon(\lambda_1\phi_1^p - |\nabla\phi_1|^p) &\leqslant \varepsilon\lambda_1\phi_1^p\\ &\leqslant \varepsilon\lambda_1\|\phi_1\|_\infty^p\\ &\leqslant \varepsilon\lambda_1\\ &\leqslant a_1\big(\frac{p-1}{p}\varepsilon^{\frac{1}{p-1}}\sigma^{\frac{p}{p-1}}\big)^{\alpha}\big(\frac{q-1}{q}\varepsilon^{\frac{1}{q-1}}\sigma^{*\frac{q}{q-1}}\big)^{\gamma}\\ &\leqslant \lambda a(x)\psi_1^{\alpha}\psi_2^{\gamma}.\end{aligned} \tag{2.11}$$

A similar argument shows that:

$$\varepsilon(\lambda_1^*\phi_1^{*q} - |\nabla\phi_1^*|^q) \leqslant \lambda b(x)\psi_1^{\eta}\psi_2^{\beta}. \tag{2.12}$$

Then we obtain from (2.3) , (2.4) and (2.11) , (2.12) that

$$\int_{\Omega-\overline{\Omega}_\delta} |\nabla\psi_1|^{p-2}|\nabla\psi_1|\cdot\nabla f_1 dx \leqslant \lambda\int_{\Omega-\overline{\Omega}_\delta} a(x){\psi_1}^{\alpha}{\psi_2}^{\gamma} f_1 dx, \tag{2.13}$$

$$\int_{\Omega-\overline{\Omega}_\delta} |\nabla\psi_2|^{q-2}|\nabla\psi_2|\cdot\nabla f_2 dx \leqslant \lambda\int_{\Omega-\overline{\Omega}_\delta} b(x){\psi_1}^{\eta}{\psi_2}^{\beta} f_2 dx. \tag{2.14}$$

Since $\Omega = \overline{\Omega}_\delta \cup (\Omega\backslash\overline{\Omega}_\delta)$, We obtain from (2.8) , (2.9) and (2.13) , (2.14) that

$$\int_{\Omega} |\nabla\psi_1|^{p-2}|\nabla\psi_1|\cdot\nabla f_1 dx \leqslant \lambda\int_{\Omega} a(x){\psi_1}^{\alpha}{\psi_2}^{\gamma} f_1 dx, \tag{2.15}$$

$$\int_{\Omega} |\nabla\psi_2|^{q-2}|\nabla\psi_2|\cdot\nabla f_2 dx \leqslant \lambda\int_{\Omega} b(x){\psi_1}^{\eta}{\psi_2}^{\beta} f_2 dx, \tag{2.16}$$

we have shown that (ψ_1,ψ_2) is sub-solution.

Now, we will construct a super-solution (z_1,z_2) of (1.1). It is clear that:

$$-\operatorname{div}(|\nabla z_1|^{p-2}\nabla z_1) = A, \quad x\in\Omega, \tag{2.17}$$

$$-\operatorname{div}(|\nabla z_2|^{q-2}\nabla z_2) = B, \quad x \in \Omega. \tag{2.18}$$

We denote

$$z_1(x) = Ae_1(x), z_2(x) = Be_2(x), \tag{2.19}$$

where the constants $A, B > 0$ are large and to be chosen later. We shall verify that is a super-solution of (1.1).

Next, since $\lambda \leqslant \overline{\lambda}$ we have $\lambda \leqslant \frac{1}{\|b\|_\infty}, \lambda \leqslant \frac{1}{\|a\|_\infty}$. Let $l_1 = \|e_1\|_\infty$, $l_2 = \|e_2\|_\infty$, since (i) hold, it is easy to prove that there exist positive large constants A, B such that([3]):

$$\begin{aligned} A^{p-1-\alpha} &\geqslant B^\gamma l_1^\alpha l_2^\gamma \\ &\geqslant \lambda \|a\|_\infty B^\gamma l_1^\alpha l_2^\gamma, \end{aligned} \tag{2.20}$$

and

$$\begin{aligned} B^{q-1-\beta} &\geqslant A^\eta l_1^\eta l_2^\beta \\ &\geqslant \lambda \|b\|_\infty A^\eta l_1^\eta l_2^\beta, \end{aligned} \tag{2.21}$$

These imply that:

$$A^{p-1} \geqslant \lambda a(x) z_1^\alpha z_2^\gamma, \quad B^{q-1} \geqslant \lambda b(x) z_1^\delta z_2^\beta \tag{2.22}$$

Let $f_1 \in W_0^{1,p}(\Omega)$, $f_2 \in W_0^{1,q}(\Omega)$ with $f_1, f_2 > 0$. Then we obtain from (2.12) , (2.13) and (2.17) that

$$\int_\Omega |\nabla z_1|^{p-2}|\nabla z_1| \cdot \nabla f_1 dx = A^{p-1} \int_\Omega f_1(x) dx \geqslant \lambda \int_\Omega a(x) {z_1}^\alpha {z_2}^\gamma f_1 dx, \tag{2.23}$$

$$\int_\Omega |\nabla z_2|^{p-2}|\nabla z_2| \cdot \nabla f_2 dx = B^{q-1} \int_\Omega f_2(x) dx \geqslant \lambda \int_\Omega a(x) {z_1}^\eta {z_2}^\beta f_2 dx. \tag{2.24}$$

a.e. in Ω. Thus, (z_1, z_2) is a super-solution of (1.1). Obviously, we have $z_i(x) \geqslant \psi_i(x)$ in Ω with large A, B for $i = 1, 2$. Thus, by Lemma 1, there exists a positive solution (u, v) of (1.1) such that $(\psi_1, \psi_2) \leqslant (u, v) \leqslant (z_1, z_2)$. This completes the proof of Theorem 1. □

REFERENCES

[1] J. ALI, R. SHIVAJI, *Positive solutions for a class of p-Laplacian systems with multiple parameters*, J. Math. Anal. Appl., 335 (2007), 1013–1019.

[2] A. CANADA, P. DRABEK, J.L. GAMES, *Existence of positive solutions for some problems with nonlinear diffusion*, Tras. Amer. Math. Soc., 349 (1997), 4231–4249.

[3] C. CHEN, *On positive weak solutions for a class of quasilinear elliptic systems*, Nonlinear Anal., 62 (2005), 751–756.

[4] M. CHHETRI, D.D. HAI, R. SHIVAJI, *On positive solutions for classes of p-Laplacian semipositone systems*, Discrete Continuous Dyn. Syst., **9**, 4 (2003), 1063–1071.

[5] F. CÌRSTEA, V. RĂDULESCU, *Entire solutions blowing up at infinity for semilinear elliptic systems*, J. Math. Pures Appl., 81 (2002), 827–846.

[6] E. ESCOBEDO, M.A. HERRERO, *Boundedness and blow-up for a semilinear reaction-diffusion system*, J. Diff. Eqns., 89 (1991), 176–202.

[7] P. GLANSDROFF, I. PRIGOGINE, *Thermodynamic Theory of Structure, Stability and Fluctuation*, Wiley Interscience, London, 1971.

[8] Z. GUO, *Some existence and multiplicity results for a class of quasilinear elliptic eigenvalue problems*, Nonlinear Anal., 18 (1992), 957–971.

[9] Z. GUO, *On the number of positive solutions for quasilinear elliptic eigenvalue problems*, Nonlinear Anal., 27, (2)(1996), 229–247.

[10] Z. GUO, *Existence and uniqueness of positive radial solutions for a class of quasilinear elliptic equations*, Applicable Anal., 47 (1992), 173–190.

[11] D.D. HAI, *On a class of quasilinear systems with sign-changing nonlinearities*, J. Math. Anal. Appl., 334 (2007), 965–976.

[12] A.V. LAIR, A.W. WOOD, *Existence of entire large positive solutions of semilinear elliptic systems*, J. Diff. Eqns., 164, (2) (2000), 380–394.

[13] H. MEINHARDT, *Models of Biological Pattern Formation*, Academic Press, London, 1982.

[14] S.H. RASOULI, Z. HALIMI, Z. MASHHADBAN, *A remark on the existence of positive weak solution for a class of* (p,q) *-Laplacian nonlinear system with sign-changing weight*, Nonliear Anal., 73 (2010), 385–389.

[15] Z.D. YANG, *Existence of entire explosive positive radial solutions for a class of quasilinear elliptic systems*, J. Math. Anal. Appl., 288 (2003), 768–783.

[16] Z. YANG, Q. LU, *Nonexistence of positive solutions to a quasilinear elliptic system and blow-up estimates for a quasilinear reaction-diffusion system*, J. Compu. Appl. Math., 150 (2003), 37–56.

[17] H. YIN, Z. YANG, *New result on the existence of bounded positive entire solutions for quasilinear elliptic systems*, Appl. Math. Comput., 190 (2007), 441–448.

[18] H. YIN, *Existence result for classes of quasilinear elliptic systems with sign-changing weight*, International J. of Non. Science., 10 (2010), 53–60.

(Received April 27, 2013)
(Revised March 28, 2014)

Dianpeng He
Institute of Mathematics, School of Mathematical Sciences
Nanjing Normal University, Jiangsu Nanjing 210023
China
e-mail: hehaipengdq@163.com

Zuodong Yang
Institute of Mathematics, School of Mathematical Sciences
Nanjing Normal University, Jiangsu Nanjing 210023
China;
School of Teacher Education
Nanjing Normal University, Jiangsu Nanjing 210097
China
e-mail: zdyang_jin@263.net