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GLOBAL EXISTENCE OF RADIAL SOLUTIONS OF A HYPERBOLIC MEMS EQUATION WITH NONLOCAL TERM

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Abstract. We consider a nonlocal hyperbolic MEMS equation in the higher dimensional annular domain. In this paper, we concentrate on the radial solutions. First we establish a time-local solution by a contraction mapping theorem. This procedure is standard. Next we show that there exists a global solution for small parameter and initial value. The important facts for the proof are the Sobolev embedding theorem and the energy conservation. Finally, we deal with the corresponding stationary problem. By the maximum principle, we can evade integrating the stationary solution over the domain near the boundary. Then we establish the upper bound of the parameter for the existence of the stationary solution.

1. Introduction

The Micro-Electro Mechanical System(MEMS) is often utilized to combine electronics with micro-size mechanical devices. The MEMS devices can be modelled as the dynamic deflection of an elastic membrane inside this system and arise in the accelerometers for airbag deployment in automobiles, in the ink jet printer heads, in the optical switches, in the chemical sensors and so on. For more details see [35] and references therein. Typically, the devices consist of an elastic membrane suspended above a rigid ground plate with a fixed voltage source and a fixed capacitor. In the case where the distance between the two plates is relative small compared to the length of the device, the original mathematical system describing the operation of the MEMS is reduced to the equation with nonlocal term. Denoting the deflection of the membrane by u, we have

$$u_{tt}+\varepsilon u_t=\Delta u+\lambda\frac{f(x)}{(1-u)^2\left(1+\alpha\int_\Omega\frac{dx}{1-u}\right)^2},\qquad x\in\Omega,\ t\in(0,T)\tag{1.1}$$

with $u(x,t)=0$ on $x\in\partial\Omega$, $t\in(0,T)$, where Ω is a bounded domain in $\mathbb{R}^2$ and $T>0$ is a maximal existing time of the solution u. Here ε is the ratio of the interaction due to the inertial and damping terms in the model, $\lambda=V^2L^2\varepsilon_0/(2\tau l^2)$ and α is the ratio of a fixed capacitance to a reference one of the device. The physical constants V, τ, L,

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l and ε_0 stand for the applied voltage, the tension in the membrane, the characteristic length of the domain Ω, the characteristic width of the gap between the membrane and the fixed electrode and the permittivity of the free space. For the derivation and related topics, see [7, 35, 36]. The function $f(x)$ represents varying dielectric properties of the membrane and applied alternating current. Physically $f(x)$ is supposed to be positive in $\overline{\Omega}$. Some typical examples [18] of dielectric profile are given as

$$f(x) = |x|^q \qquad \text{for} \qquad q > 0$$

and

$$f(x) = e^{k\left(|x|^2 - c\right)} \qquad \text{for} \qquad k, c > 0.$$

The nonlocal term in (1.1) arises due to the fact that the device is embedded in an electrical circuit with a capacitor of fixed capacitance. In the limiting case $\alpha = 0$, there is supposed to be no capacitor in the circuit. It is also assumed that the edges of the membrane are kept fixed leading to Dirichlet boundary conditions, whereas it is usually considered that initially the elastic membrane is in rest corresponding to $u(x,0) \equiv 0$. In this paper, we treat the radial problem, where Ω is an annular domain $A_a \equiv \{x \in \mathbb{R}^n \mid a < |x| < 1\}$ for $n \geqslant 2$ with $0 < a < 1$, $f(x) \equiv 1$, $u(x,0) = u_0(x)$, $u_t(x,0) = v_0(x)$ and $u_0 \in [0,1)$. Here, $u_0(x)$ and $v_0(x)$ are supposed to belong to an appropriate function space. We consider the case where the u_t term is much smaller than the u_{tt} and Δu terms in (1.1). Then (1.1) with $\alpha = 1$ is reduced to the following nonlocal hyperbolic problem:

$$\begin{cases} u_{tt} = \Delta u + \lambda \dfrac{1}{(1-u)^2\left(1+\int_\Omega \frac{dx}{1-u}\right)^p}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) \in [0,1), & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.2}$$

where $\lambda > 0$, $p > 1$ and $T > 0$. If the solution $u(x,t)$ of (1.1) reaches 1 at some point in Ω in finite time $t = T_q$, the right-hand side of (1.1) becomes infinite, which leads to the singularity. In this case, the solution $u(x,t)$ is said to quench in finite time $t = T_q$ and T_q is called the quenching time of the solution. The quenching behaviour physically corresponds to the phenomenon of "touch-down" i.e., the elastic membrane touches the ground electrode. In applications, the touch-down phenomenon is observed when the applied voltage V at the ends of the electrical circuit exceeds a fixed value. In [23], Kavallaris, Lacey, Nikolopoulos and Tzanetis consider (1.2) for $\Omega = (0,1)$, $p = 2$ and $f(x) = 1$. They obtain the global existence and quenching results of the solution for sufficiently small and large $\lambda > 0$, respectively. Lately, in [13], Guo and Huang consider the damped hyperbolic equation (1.1) for $\Omega = (0,1)$ and $f(x) = 1$ and obtain the results similar to those in [23]. Their key facts are the one-dimensional Sobolev embedding $H^1(\Omega) \subset C(\overline{\Omega})$ and the one-dimensional representation formula for the elementary solution of the wave equation. Hence their ideas are not applicable to the domain for higher dimension, which is our motivation of this paper. In [28, 29], Liang, Li and Zhang consider the damped hyperbolic equation (1.1) for higher dimensional domain. We introduce their interesting results of the global existence, quenching and stability in Section 5 and compare our results to theirs. For more details, see Section 5. If we consider the case $\alpha = 0$ i.e.,

$$\begin{cases} u_{tt} = \Delta u + \lambda \frac{1}{(1-u)^{\beta}}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) \in [0,1), & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{1.3}$$

there are many results for global existence of the solution for sufficiently small $\lambda > 0$ (e.g. see [3] for $\beta = 1$ and [26, 38] for an abstract nonlinearity), quenching results for sufficiently large $\lambda > 0$ (e.g. see [3] for $\beta = 1$ and [26, 33, 38] for an abstract nonlinearity), the estimate of the quenching time [33] and the singularity of the derivative (e.g. see [2] for an abstract nonlinearity). In stead of (1.2), we suppose that the u_t term dominates the u_{tt} term in (1.1). Then we have the following nonlocal parabolic equation:

$$\begin{cases} u_t = \Delta u + \lambda \frac{1}{(1-u)^2\left(1+\int_\Omega \frac{dx}{1-u}\right)^p}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) \in [0,1), & x \in \Omega. \end{cases}$$

This problem has been studied by many authors (e.g. see [12, 14, 20, 34, 36] for $p = 2$). They obtain the results similar to those for (1.3). In [20] Hui and in [14] Guo and Kavallaris show that the global solution converges to the stationary minimal solution of local problem

$$\begin{cases} \Delta u + \lambda \frac{1}{(1-u)^2} = 0, & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega \end{cases} \tag{1.4}$$

for sufficiently small $\lambda > 0$, respectively. In [14], they investigate the structure of the set of stationary solution

$$\begin{cases} \Delta u + \lambda \frac{1}{(1-u)^2\left(1+\int_\Omega \frac{dx}{1-u}\right)^p} = 0, & x \in \Omega, \\ u(x) = 0, & x \in \partial\Omega \end{cases} \tag{1.5}$$

and its spectral properties. Also in the parabolic equation, the local problem

$$\begin{cases} u_t = \Delta u + \lambda \frac{1}{(1-u)^{\beta}}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) \in [0,1), & x \in \Omega \end{cases} \tag{1.6}$$

has been considered. Thanks to the maximum principle, we have the results of global existence (e.g. see [25] for general β and [9] for $\beta = 2$), quenching (e.g. see [9, 16, 17] for $\beta = 2$ and [24, 25] for general β), the connecting orbit [24], the Morse-Smale property [24] and its stationary solution (e.g. see [11, 24] for general β and [4, 5, 6, 8] for $\beta = 2$). In the previous works, Ω has been supposed to be a unit ball or convex domain. The aim of this paper is to consider (1.2) in the case of $p > 1$ and annular domain. In this paper, we treat the radial solution $u = u(|x|,t) = u(r,t)$ of (1.2) with $r = |x|$ and concentrate on the following equation:

$$\begin{cases} u_{tt} = u_{rr} + \frac{n-1}{r} u_r + \lambda \frac{1}{(1-u)^2 \left(1+\omega_n \int_a^1 \frac{r^{n-1}}{1-u} dr\right)^p}, & r \in I,\ t \in (0,T), \\ u(a,t) = u(1,t) = 0, & t \in (0,T), \\ u(r,0) = u_0(r) \in [0,1), & r \in I, \\ u_t(r,0) = v_0(r), & r \in I, \end{cases} \tag{1.7}$$

where $n \geqslant 2$, $p > 1$, $I \equiv (a,1)$ and ω_n denotes the area of the unit sphere in $\mathbb{R}^n$. The first theorem is concerned with the local existence of the solution.

THEOREM 1. *Let $D \equiv H_0^1(I) \times L^2(I)$ and $H \equiv L^2(I) \times H^{-1}(I)$. For any $n \geqslant 2$, $p > 1$, $\lambda > 0$ and $(u_0, v_0) \in D$ with*

$$\|u_0\|_{C(\bar{I})} \leqslant 1 - \delta$$

for $0 < \delta < 1$, there exists a unique solution of (1.7) with

$$\phi = \begin{pmatrix} u \\ u_t \end{pmatrix} \in C\big([0,T);D\big) \cap C^1\big([0,T);H\big)$$

for sufficiently small $T > 0$, where $\|\cdot\|_{C(\bar{I})}$ denotes the norm of the space of continuous functions in $\bar{I}$. The solution u can be continued as long as $\max_{r \in \bar{I}} u(r,t) < 1$.

Throughout this paper, the definition of the function spaces and norms is presented in Section 2. In the second theorem, we derive the global existence of the solution. We define the energy $\mathscr{E}_0$ of initial function by

$$\mathscr{E}_0 = \frac{1}{2}\left(\|u_0\|^2_{H_0^1(I)} + \|v_0\|^2_{L^2(I)} \right).$$

THEOREM 2. *Let $D \equiv H_0^1(I) \times L^2(I)$ and $H \equiv L^2(I) \times H^{-1}(I)$. We assume that $n \geqslant 2$ and $p > 1$. For any $(u_0, v_0) \in D$ with*

$$\|u_0\|_{C(\bar{I})} \leqslant 1 - \delta$$

for $0 < \delta < 1$, small parameter

$$\lambda < \frac{\omega_n (p-1) a^{n-1}}{1-a} \left(1 + \frac{1}{n}(1-a^n)\omega_n\right)^{p-1}$$

and small initial functions

$$\mathscr{E}_0 \leqslant \frac{a^{n-1}}{1-a} - \frac{\lambda}{\omega_n (p-1)} \left(\frac{1}{1 + \frac{1}{n}(1-a^n)\omega_n} \right)^{p-1} - \theta$$

for some small $\theta > 0$, *there exists a unique global solution of* (1.7) *with*

$$\phi = \begin{pmatrix} u \\ u_t \end{pmatrix} \in C\big([0,\infty);D\big) \cap C^1\big([0,\infty);H\big).$$

For the higher dimensional general domain, we can show that the solution of (1.3) quenches in finite time for sufficiently large $\lambda > 0$ as proven in [33, 38], while it remains open for (1.2). For example, one of the reasons is that it is difficult to investigate the location of quenching points. Lately, in [15], Guo and Souplet derive the conditions on β and $f(x)$ under which the quenching point of the solution of

$$\begin{cases} u_t = \Delta u + \lambda \frac{f(x)}{(1-u)^\beta}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = 0, & x \in \Omega \end{cases}$$

is not near the boundary $\partial\Omega$. Since the proof is based heavily on the maximum principle, it is not applicable to our problem. Finally we consider the following stationary problem:

$$\begin{cases} \Delta u + \lambda \frac{1}{(1-u)^2\left(1+\int_{A_a} \frac{dx}{1-u}\right)^p} = 0, & x \in A_a, \\ u(x) = 0, & x \in \partial A_a. \end{cases} \tag{1.8}$$

For fixed $\lambda > 0$, we define the section of radial solution set by

$$\mathscr{C}^\lambda = \{u \mid u = u(|x|) = u(r) \text{ is a classical solution of (1.8) for } \lambda > 0\}.$$

Then the third theorem is on the existence of the solution of (1.8).

THEOREM 3. *We assume that* $n = 2$ *and* $1 < p \leqslant 2$. *Then there is* $\overline{\lambda} \in (0,+\infty)$ *such that* $\mathscr{C}^\lambda = \emptyset$ *for all* $\lambda > \overline{\lambda}$.

This paper is organized as follows: In Section 2, we transform (1.7) to the integral equation and apply the contraction mapping theorem to it so that we can obtain the local solution. In Section 3, we consider the radial global solution for small parameter λ and initial value (u_0, v_0). In Section 4, we obtain the upper bound of λ for the existence of the stationary solution. In Section 5, we compare our results in this paper to those for the damped equations obtained by [28, 29]. We discuss the advantage of the damping term.

2. Local solution

We transform (1.7) to the modified Schrödinger equation by [37, 38]. We apply the contraction mapping theorem owing to the facts for the Schrödinger equation in [1]. Then we establish the local solution. This is a standard manner to prove. In this paper, $C(\overline{I})$ denotes the space of continuous functions in $\overline{I}$ with the norm

$$\|w\|_{C(\overline{I})} = \sup_{r \in \overline{I}} |w(r)|$$

for $w \in C(\bar{I})$ and $H^s(I)$ denotes the usual Sobolev space in I with the norm

$$\|w\|_{H^s(I)} = \left(\sum_{k=0}^{s} \left\| \frac{\partial^k}{\partial r^k} w \right\|_{L^2(I)}^2 \right)^{\frac{1}{2}}$$

for $w \in H^s(I)$. Here, $\|\cdot\|_{L^p(I)}$ denotes the standard L^p norm in I. $H_0^s(I)$ is defined as the closure of the set $\mathscr{D}(I)$ in the space $H^s(I)$, where we denote by $\mathscr{D}(I)$ the space of all infinitely differentiable functions on I with compact supports. Now the following Poincaré inequality holds:

$$\|w\|_{L^2(I)} \leqslant C_P \|w_r\|_{L^2(I)}$$

for $w \in H_0^1(I)$ for some constant $C_P > 0$ depending only on I. Hence we adopt the norm in $H_0^1(I)$ as

$$\|w\|_{H_0^1(I)} = \|w_r\|_{L^2(I)}.$$

$H^{-s}(I)$ is defined as the dual space of $H_0^s(I)$ with the norm

$$\|w\|_{H^{-s}(I)} = \sup_{\phi \in H_0^s(I), \|\phi\|_{H_0^s(I)} \leqslant 1} \left| \int_I w(r)\phi(r)dr \right|$$

for $w \in H^{-s}(I)$. Since $I \subset \mathbb{R}$, we have the Sobolev embedding inequality

$$\|w\|_{C(\bar{I})} \leqslant C_S \|w\|_{H_0^1(I)}$$

for $w \in H_0^1(I)$ for some constant $C_S > 0$ depending only on I. Note that we can take $C_S = \sqrt{(1-a)/2}$. For the homogeneous wave equation

$$\begin{cases} u_{tt} = u_{rr}, & r \in I,\ t > 0, \\ u(a,t) = u(1,t) = 0, & t > 0, \\ u(r,0) = u_0(r), & r \in I, \\ u_t(r,0) = v_0(r), & r \in I, \end{cases} \tag{2.1}$$

we define

$$\phi = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u \\ u_t \end{pmatrix}, \quad A = i \begin{pmatrix} 0 & I \\ -B^2 & 0 \end{pmatrix},$$

where $i = \sqrt{-1}$, $B^2 = -\partial^2/\partial r^2$ is a positive definite self-adjoint operator of $H^{-1}(I)$ with domains $\mathscr{D}(B^2) = H_0^1(I)$, $\mathscr{D}(A) = D \subset H$ and I denotes the identity operator on $L^2(I)$. Then, we can write (2.1) into the homogeneous Schrödinger equation

$$\begin{cases} \phi_t = -iA\phi, & r \in I,\ t > 0, \\ \phi(a,t) = \phi(1,t) = 0, & t > 0, \\ \phi(r,0) = \phi_0(r) = \begin{pmatrix} u_0(r) \\ v_0(r) \end{pmatrix}, & r \in I. \end{cases} \tag{2.2}$$

Then we introduce the following well-known theorem.

LEMMA 1. (Théorème X.14 in [1]) *For any* $\phi_0 \in D$*, there exists a unique solution*

$$\phi \in C\big([0,+\infty);D\big) \cap C^1\big([0,+\infty);H\big)$$

of (2.2)*. Moreover, we have*

$$\|\phi(\,\cdot\,,t)\|_D = \|\phi_0\|_D$$

for $t > 0$*, where*

$$\|\phi\|_D = \|u\|_{H_0^1(I)} + \|v\|_{L^2(I)}\,.$$

We denote the mapping $e^{-iAt} : D \to D$ by

$$e^{-iAt}\phi_0(\,\cdot\,) = \phi(\,\cdot\,,t).$$

First of all, let

$$F(u) = \frac{1}{1-u} \qquad \text{and} \qquad G(u) = 1 + \omega_n \int_I F(u(r,t)) r^{n-1} dr.$$

For $0 < \delta < 1$, we define the modification of F by

$$F_\delta(u) = \begin{cases} F(u), & u \leqslant 1-\delta, \\ F(1-\frac{\delta}{2}), & u \geqslant 1-\frac{\delta}{2}. \end{cases}$$

Here we define F_δ suitably in the range $(1-\delta, 1-\delta/2)$ so that we assume that F_δ is positive, bounded and uniformly Lipschitz continuous on $\mathbb{R}$. Putting

$$G_\delta(u) = 1 + \omega_n \int_I F_\delta(u(r,t)) r^{n-1} dr,$$

we define

$$J(\phi) = \begin{pmatrix} 0 \\ \frac{n-1}{r} u_r + \frac{\lambda F(u)^2}{G(u)^p} \end{pmatrix} \qquad \text{and} \qquad J_\delta(\phi) = \begin{pmatrix} 0 \\ \frac{n-1}{r} u_r + \frac{\lambda F_\delta(u)^2}{G_\delta(u)^p} \end{pmatrix}.$$

Under these notations, we have the integral equation corresponding to (1.7)

$$\phi = e^{-iAt}\phi_0 + \int_0^t e^{-iA(t-s)} J(\phi(s)) ds \tag{2.3}$$

and the modified integral equation with the same initial function ϕ_0

$$\phi = e^{-iAt}\phi_0 + \int_0^t e^{-iA(t-s)} J_\delta(\phi(s)) ds. \tag{2.4}$$

From now on, we concentrate on (2.4). Taking $\eta = \|\phi_0\|_D \equiv \|u_0\|_{H_0^1(I)} + \|v_0\|_{L^2(I)}$, we set

$$X_T \equiv \left\{ \phi \in C\big([0,T];D\big) \mid \|\phi\|_{X_T} \leqslant 2\eta \right\},$$

where T is a positive constant to be determined later. Here in the space X_T, the norm is equipped with

$$\|\phi\|_{X_T} = \sup_{t\in[0,T]} \|\phi(\,\cdot\,,t)\|_D = \sup_{t\in[0,T]} \Big(\|u(\,\cdot\,,t)\|_{H_0^1(I)} + \|v(\,\cdot\,,t)\|_{L^2(I)} \Big).$$

For $\phi \in X_T$, we define the mapping $S(t)$ from D by the right-hand side of (2.4), that is,

$$S\phi = e^{-iAt}\phi_0 + \int_0^t e^{-iA(t-s)} J_\delta(\phi(s))ds.$$

Then we show that S is a contraction mapping from X_T into X_T for sufficiently small $T>0$.

LEMMA 2. *If*

$$T < T_1 \equiv \frac{\eta}{\frac{2(n-1)\eta}{a} + \lambda C_1^2\sqrt{1-a}},$$

then S is a mapping from X_T into X_T, where $C_1 = \|F_\delta\|_{C(\mathbb{R})}$.

Proof. Let $\phi = \begin{pmatrix} u \\ v \end{pmatrix} \in X_T$. Since $1 \leqslant G_\delta(s)$ holds for all $s\in\mathbb{R}$, we have

$$\begin{aligned}
\|S\phi\|_D &\leqslant \left\|e^{-iAt}\phi_0\right\|_D + \int_0^t \left\|e^{-iA(t-s)}J_\delta(\phi(s))\right\|_D ds \\
&= \|\phi_0\|_D + \int_0^t \|J_\delta(\phi(s))\|_D ds \\
&\leqslant \eta + \int_0^t \left\|\frac{n-1}{r}u_r\right\|_{L^2(I)} ds + \lambda\int_0^t \frac{\left\|F_\delta(u)^2\right\|_{L^2(I)}}{G_\delta(u)^p} ds \\
&\leqslant \eta + \left(\frac{n-1}{a}\|\phi\|_{X_T} + \lambda C_1^2|I|^{\frac{1}{2}}\right)T \\
&\leqslant \eta + \left(\frac{2(n-1)\eta}{a} + \lambda C_1^2|I|^{\frac{1}{2}}\right)T \\
&\leqslant 2\eta
\end{aligned}$$

and

$$\|S\phi\|_{X_T} \leqslant 2\eta,$$

where $|I|$ denotes the measure $1-a$ of I in $\mathbb{R}$. $\square$

LEMMA 3. *If*

$$T < T_2 \equiv \min\left(T_1, \frac{1}{2}\frac{1}{\frac{n-1}{a} + \lambda C_1\sqrt{1-a}(2LC_S + MC_1)}\right),$$

then S is a contraction mapping from X_T into X_T. Here $C_S > 0$ is an embedding constant depending only on I and M is a positive constant depending only on p, n, I, C_1 and Lipschitz constant L of $F_\delta(s)$ on $\mathbb{R}$, respectively.

Proof. Let $\phi = \begin{pmatrix} u_1 \\ v_1 \end{pmatrix}, \psi = \begin{pmatrix} u_2 \\ v_2 \end{pmatrix} \in X_T$. Setting

$$C_2 = \left(1 + \omega_n C_1 |I|\right)^{p-1},$$

we note

$$\begin{aligned} |G_\delta(u_1)^p - G_\delta(u_2)^p| &\leqslant p \max_{u\in\mathbb{R}} G_\delta(u)^{p-1} |G_\delta(u_1) - G_\delta(u_2)| \\ &\leqslant p\omega_n C_2 \int_I |F_\delta(u_1) - F_\delta(u_2)|\, dr \\ &\leqslant p\omega_n C_2 |I| LC_S \sup_{t\in[0,T]} \|u_1 - u_2\|_{H_0^1(I)} \\ &\leqslant M \|\phi - \psi\|_{X_T}, \end{aligned}$$

where $M = p\omega_n C_2 |I| LC_S$. We have

$$\begin{aligned} &\|S\phi - S\psi\|_D \\ &\leqslant \int_0^t \left\| e^{-iA(t-s)} \Big(J_\delta(\phi(s)) - J_\delta(\psi(s)) \Big) \right\|_D ds \\ &\leqslant \int_0^t \left\| \frac{n-1}{r} (u_1 - u_2)_r \right\|_{L^2(I)} ds + \lambda \int_0^t \left\| \frac{F_\delta(u_1)^2}{G_\delta(u_1)^p} - \frac{F_\delta(u_2)^2}{G_\delta(u_2)^p} \right\|_{L^2(I)} ds \\ &\leqslant \frac{n-1}{a} \|\phi - \psi\|_{X_T} T + \lambda \int_0^t \left\| \frac{F_\delta(u_1)^2 - F_\delta(u_2)^2}{G_\delta(u_1)^p} \right\|_{L^2(I)} ds \\ &\quad + \lambda \int_0^t \left\| F_\delta(u_2)^2 \frac{G_\delta(u_1)^p - G_\delta(u_2)^p}{G_\delta(u_1)^p G_\delta(u_2)^p} \right\|_{L^2(I)} ds \\ &\leqslant \left(\frac{n-1}{a} + 2\lambda C_1 C_S L |I|^{\frac{1}{2}} + \lambda C_1^2 M |I|^{\frac{1}{2}} \right) \|\phi - \psi\|_{X_T} T \\ &\leqslant \frac{1}{2} \|\phi - \psi\|_{X_T} \end{aligned}$$

and

$$\|S\phi - S\psi\|_{X_T} \leqslant \frac{1}{2} \|\phi - \psi\|_{X_T}$$

for $T < T_2$. □

Proof of Theorem 1. By Lemmas 2 and 3, the mapping S is a contraction from X_T to X_T for sufficiently small $T \in (0, T_2)$. Hence (2.4) has a unique time local solution $\phi \in C\big([0,T);D\big)$. Since we get

$$\begin{aligned} \|v_t\|_{H^{-1}(I)} &\leqslant \|u_{rr}\|_{H^{-1}(I)} + (n-1) \left\| \frac{u_r}{r} \right\|_{H^{-1}(I)} + \lambda \left\| \frac{F_\delta(u)^2}{G_\delta(u)^p} \right\|_{H^{-1}(I)} \\ &\leqslant \left(1 + \frac{n-1}{a} C_P\right) \|u\|_{H_0^1(I)} + \lambda C_P \left\| \frac{F_\delta(u)^2}{G_\delta(u)^p} \right\|_{L^2(I)} \end{aligned}$$

$$\leqslant \left(1+\frac{n-1}{a}C_P\right)\|u\|_{H_0^1(I)}+\lambda C_P|I|^{\frac{1}{2}}C_1^2,$$

$\phi_t \in C\big([0,T);H\big)$ follows. If the solution of (2.4) begins with $0 \leqslant u_0(r) \leqslant 1-\delta$ and satisfies $\max_{r\in\bar{I}} u(r,t) \leqslant 1-\delta$ for all $t>0$, then u is a solution of (2.3) and hence (1.7). Otherwise there is a finite time $T_0>0$ at which $\max_{r\in\bar{I}} u(r,T_0)=1-\delta$. We choose $\delta_1 \in (0,\delta)$ and apply the contraction mapping theorem to (2.4) with δ replaced by δ_1. We may extend $u(r,t)$ uniquely to an interval $(0,T_0')$ with $T_0<T_0'$ such that $\max_{r\in\bar{I}} u(r,t) \leqslant 1-\delta_1$ for $[0,T_0')$. Since we can take $\delta_1 \in (0,\delta)$ arbitrarily small, $u(r,t)$ is a solution of (1.7) on $I\times[0,T_0')$ as long as $\max_{r\in\bar{I}} u(r,t)<1$. □

3. Global existence

In the use of the conserved energy, we extend the local solution obtained in Theorem 1 globally in time. We note that the inclusion $H_0^1(I)\subset C(\bar{I})$ with an embedding constant $C_S=\sqrt{(1-a)/2}$, where $I=(a,1)$. Then we obtain

$$0 \leqslant \max_{r\in\bar{I}} u(r,t) \leqslant \|u(t)\|_{C(\bar{I})} \leqslant \sqrt{\frac{1-a}{2}}\,\|u(t)\|_{H_0^1(I)}$$

for $t\in[0,T_q)$, where T_q is the quenching time. We follow the procedure in the proof given in [23]. First we define

$$g(x)=\alpha x^2+\beta\left(\frac{1-x}{\gamma-x}\right)^{p-1}$$

for $0\leqslant x\leqslant 1$ and

$$\alpha=\frac{a^{n-1}}{1-a},\quad \beta=\frac{\lambda}{\omega_n(p-1)}\quad\text{and}\quad \gamma=1+\frac{1}{n}(1-a^n)\omega_n.$$

Proof of Theorem 2. Since we have established the local solution, we have only to derive the a priori estimate. Then defining a conserved energy by

$$E(t)=\frac{1}{2}\int_a^1(u_t^2+u_r^2)r^{n-1}dr+\beta\frac{1}{\left(1+\omega_n\int_a^1\frac{r^{n-1}}{1-u}dr\right)^{p-1}},$$

we get $E'(t)=0$. Now let $m(t)=\max_{r\in\bar{I}} u(r,t)$. We have

$$\begin{aligned}E(t) &\geqslant \frac{a^{n-1}}{2}\|u\|_{H_0^1(I)}^2+\beta\frac{1}{\left(1+\omega_n\int_a^1\frac{r^{n-1}}{1-m(t)}dr\right)^{p-1}}\\ &\geqslant \frac{a^{n-1}}{1-a}m(t)^2+\beta\frac{1}{\left(1+\omega_n\frac{\frac{1}{n}(1-a^n)}{1-m(t)}\right)^{p-1}}\end{aligned}$$

$$= \alpha m(t)^2 + \beta \left(\frac{1-m(t)}{1+\frac{1}{n}(1-a^n)\omega_n - m(t)} \right)^{p-1}$$
$$= g(m(t)).$$

On the other hand, we obtain

$$\begin{aligned} E(0) &\leqslant \frac{1}{2}\Big(\|u_0\|^2_{H_0^1(I)} + \|v_0\|^2_{L^2(I)} \Big) + \beta \frac{1}{\left(1+\omega_n \int_a^1 \frac{r^{n-1}}{1-u_0} dr\right)^{p-1}} \\ &\leqslant \mathscr{E}_0 + \beta \frac{1}{\left(1+\omega_n \int_a^1 r^{n-1} dr\right)^{p-1}} \\ &= \mathscr{E}_0 + \beta \frac{1}{\left(1+\omega_n \frac{1-a^n}{n}\right)^{p-1}} \\ &= \mathscr{E}_0 + g(0). \end{aligned}$$

Since $E(t) = E(0)$ holds, we have

$$g(m(t)) \leqslant \mathscr{E}_0 + g(0) \leqslant g(1) - g(0) - \theta + g(0) = g(1) - \theta. \tag{3.1}$$

Now letting

$$m(t) \to 1-0$$

as $t \to T_q$, then we obtain $\theta \leqslant 0$ from (3.1). However this is a contradiction. Hence we conclude that

$$m(t) \leqslant 1 - \delta_2$$

for some $\delta_2 \in (0,1)$. Owing to the conserved energy, we have

$$\|\phi\|_D = \|u\|_{H_0^1(I)} + \|u_t\|_{L^2(I)} < 2\sqrt{\frac{2}{a^{n-1}}E(0)} < +\infty,$$

which implies that $\phi \in C\big([0,+\infty);D\big)$. In the same manner as the proof of Theorem 1, we obtain

$$\begin{aligned} \|\phi_t\|_H &= \|u_t\|_{L^2(I)} + \|v_t\|_{H^{-1}(I)} \\ &< \left(2 + \frac{n-1}{a}C_P\right)\sqrt{\frac{2}{a^{n-1}}E(0)} + \lambda C_P |I|^{\frac{1}{2}} \left(\frac{1}{1-m(t)}\right)^2 \\ &< +\infty, \end{aligned}$$

which leads us to $\phi \in C^1\big([0,\infty);H\big)$. $\square$

4. Stationary solution

If Ω is a unit ball, according to [10], any solution of (1.4) is radially symmetric. Then the solution set is investigated in [6, 21, 24]. Setting

$$\sigma = \frac{\lambda}{K} \qquad \text{and} \qquad K = \left(1+\omega_n \int_a^1 \frac{r^{n-1}}{1-u} dr\right)^p,$$

we obtain the solution set of (1.5) by means of [14, 30]. Hence, we get the radial solution of (1.5) for sufficiently small $\lambda > 0$. With this result, we utilize the argument in [31] to guarantee the existence of radial solution in an annular domain for sufficiently small $\lambda > 0$. As proven in [27], it is possible that the non-radial solution bifurcates at the point on the branch of radial solution of (1.8). However it seems to be open now. The reason is that the algebraic property of e^u plays an important role in the arguments of [27, 31, 32]. For this purpose, first of all, we have to deal with the bifurcation diagram of solution set of (1.4) for an annular domain and study its spectral properties [31, 32]. We have denoted the radial stationary solution by $u = u(r)$. In this section, we regard this solution as a function of x, denoted by $w = w(x)$. We establish the upper bound of λ for the existence of radial solution of (1.8). We replace the integral over the domain A_a by the integral over the domain ω inside the annulus A_a. The idea is based on Proposition 2.3 in [20].

PROPOSITION 1. *There exists $\tau \in (0,(1-a)/4)$ such that*

$$\int_{A_a} \frac{dx}{1-w} \leqslant C_\tau \int_{A_a \setminus \Omega_1} \frac{dx}{1-w},$$

where

$$\Omega_1 = \left\{x \in \mathbb{R}^2 \mid a < |x| < a+\tau\right\} \cup \left\{x \in \mathbb{R}^2 \mid 1-\tau < |x| < 1\right\}$$

and $C_\tau > 0$ depends only on τ. Here τ is independent of λ and the solution w.

Proof. Let $\nu = \nu(x)$ be the outer unit normal vector to $x \in \partial A_a$ with $|x| = 1$, where $|x| = \sqrt{x \cdot x}$ and $\cdot$ denotes the inner product in $\mathbb{R}^2$. We define

$$F(s) = w(\nu - s\nu)$$

for $0 \leqslant s \leqslant 1-a$. By Theorem 2.1 in [10], there exists $\tau \in (0,(1-a)/4)$ such that $F(s_1) < F(s_2)$ for any s_1, s_2 with $0 < s_1 < \tau < s_2 < 2\tau$. Because the union of the corresponding maximal cap Σ_ν to all directions ν is given by

$$\cup_\nu \Sigma_\nu = \left\{x = (x_1,x_2) \in A_a \mid \frac{a+1}{2} < |x| < 1\right\},$$

τ is independent of ν, λ and the solution w. Let

$$\Omega_{1,\tau} = \left\{x \in \mathbb{R}^2 \mid 1-\tau < |x| < 1\right\}$$

and

$$\Omega_{1,2\tau} = \left\{x \in \mathbb{R}^2 \mid 1-2\tau < |x| < 1-\tau\right\}.$$

Now that we concentrate on the radial case, we have

$$w(x) < w(y)$$

and eventually

$$\frac{1}{1-w(x)} < \frac{1}{1-w(y)}$$

for any $x \in \Omega_{1,\tau}$ and $y \in \Omega_{1,2\tau}$. Integrating it over $\Omega_{1,\tau}$ with respect to x and next $\Omega_{1,2\tau}$ with respect to y, respectively, we have

$$\int_{\Omega_{1,\tau}} \frac{dx}{1-w(x)} \leqslant \frac{|\Omega_{1,\tau}|}{|\Omega_{1,2\tau}|} \int_{\Omega_{1,2\tau}} \frac{dy}{1-w(y)} = \frac{2-\tau}{2-3\tau} \int_{\Omega_{1,2\tau}} \frac{dy}{1-w(y)},$$

where $|A|$ denotes the measure of the subset $A \subset \mathbb{R}^2$. Let

$$\Omega_{2,\tau} = \left\{x \in \mathbb{R}^2 \mid a < |x| < a+\tau\right\}$$

and

$$\Omega_{2,2\tau} = \left\{x \in \mathbb{R}^2 \mid a+\tau < |x| < a+2\tau\right\}.$$

In the case of $\Omega_{2,\tau}$ and $\Omega_{2,2\tau}$, for instance, we fix the point $x_0 = (a,0) \in \partial A_a$ and the closed disk $D = \left\{x \in \mathbb{R}^2 \mid |x-p| \leqslant a-b\right\}$, where we take $p = (b,0)$ so that $D \subset \left\{x \in \mathbb{R}^2 \mid |x| \leqslant a\right\}$ and $D \cap \overline{A}_a = \{x_0\}$. Then we adopt the Kelvin transform [10, 19, 39] given by

$$y = p + \frac{(a-b)^2}{|x-p|^2}(x-p) \qquad \text{and} \qquad z(y) = w(x).$$

The image $\tilde{A}_a$ of A_a by the transformation is included in D and touches the boundary ∂A_a only at x_0. Then since $(a-b)^{-4}|y-p|^4 \Delta_y z = \Delta_x w$ and $\tilde{A}_a$ is uniformly convex near x_0, we apply the moving plane argument to $\tilde{A}_a$. Hence taking smaller $\tilde{\tau}$ and τ if necessary, we have

$$\int_{\Omega_{2,\tau}} \frac{dx}{1-w(x)} \leqslant \frac{|\Omega_{2,\tau}|}{|\Omega_{2,2\tau}|} \int_{\Omega_{2,2\tau}} \frac{dy}{1-w(y)} = \frac{2a+\tau}{2a+3\tau} \int_{\Omega_{2,2\tau}} \frac{dy}{1-w(y)}.$$

In combination with

$$\frac{2a+\tau}{2a+3\tau} < \frac{2-\tau}{2-3\tau},$$

we have

$$\int_{\Omega_1} \frac{dx}{1-w(x)} \leqslant \frac{2-\tau}{2-3\tau} \int_{\Omega_{1,2\tau} \cup \Omega_{2,2\tau}} \frac{dy}{1-w(y)} \leqslant \frac{2-\tau}{2-3\tau} \int_{A_a \setminus \Omega_1} \frac{dy}{1-w(y)}$$

and finally

$$\int_{A_a} \frac{dx}{1-w(x)} \leqslant \frac{4-4\tau}{2-3\tau} \int_{A_a \setminus \Omega_1} \frac{dy}{1-w(y)}. \qquad \square$$

Proof of Theorem 3. We suppose that $w(x) \in \mathscr{C}^{\lambda}$ for some $\lambda > 0$. Let ϕ be the first eigenfunction of $-\Delta$ associated with first eigenvalue $\mu > 0$. Namely we have

$$\begin{cases} \Delta\phi = -\mu\phi, & x \in A_a, \\ \phi > 0, & x \in A_a, \\ \phi = 0, & x \in \partial A_a. \end{cases}$$

Here we normalize ϕ as

$$\int_{A_a} \phi dx = 1.$$

We have

$$\lambda \int_{A_a} \frac{\phi}{(1-w)^2\left(1+\int_{A_a}\frac{dx}{1-w}\right)^p} dx = -\int_{A_a} \Delta w \phi dx = \mu \int_{A_a} w\phi dx \leqslant \mu$$

by $0 \leqslant w < 1$. On the other hand, choosing $\omega = A_a \setminus \Omega_1$ and $C_3 = C_\tau$ in the proposition, we have

$$\begin{aligned}
&\lambda \int_{A_a} \frac{\phi}{(1-w)^2\left(1+\int_{A_a}\frac{dx}{1-w}\right)^p} dx \\
&\geqslant \lambda \int_{A_a} \frac{\phi dx}{(1-w)^2} \frac{1}{\left(1+C_3\int_{\omega}\frac{dx}{1-w}\right)^p} \\
&\geqslant \lambda \int_{A_a} \frac{\phi dx}{(1-w)^2} \frac{1}{\left(1+C_3\sqrt{\int_{\omega}\frac{1}{\phi}dx}\sqrt{\int_{\omega}\frac{\phi}{(1-w)^2}dx}\right)^p} \\
&\geqslant \lambda \int_{A_a} \frac{\phi dx}{(1-w)^2} \frac{1}{\left(1+\frac{C_3|\omega|^{\frac{1}{2}}}{\sqrt{\min_{x\in\omega}\phi(x)}}\sqrt{\int_{A_a}\frac{\phi}{(1-w)^2}dx}\right)^p}.
\end{aligned}$$

Hence these inequalities yield

$$\lambda \int_{A_a} \frac{\phi dx}{(1-w)^2} \frac{1}{\left(1+\frac{C_3|\omega|^{\frac{1}{2}}}{\sqrt{\min_{x\in\omega}\phi(x)}}\sqrt{\int_{A_a}\frac{\phi}{(1-w)^2}dx}\right)^p} \leqslant \mu.$$

Setting

$$C_4 = \frac{C_3\,|\omega|^{\frac{1}{2}}}{\sqrt{\min_{x\in\omega}\phi(x)}} \qquad \text{and} \qquad t = \int_{A_a} \frac{\phi dx}{(1-w)^2},$$

we obtain $0 < C_4 < +\infty$ and $t \geqslant 1$ because of $\omega \subset A_a$ and $\omega \cap \partial A_a = \emptyset$. Finally we obtain

$$\lambda \leqslant \mu \frac{\left(1+C_4\sqrt{t}\right)^p}{t}$$

and consider the behaviour of the function

$$h(t) = \frac{\left(1+C_4\sqrt{t}\right)^p}{t}$$

for $t \geqslant 1$. Because of $h(1) = (1+C_4)^p$ and

$$h'(t) = \frac{\left(1+C_4\sqrt{t}\right)^{p-1}}{t^2}\left(C_4\left(\frac{p}{2}-1\right)\sqrt{t}-1\right) < 0$$

for $t \geqslant 1$, we have $h(t) \leqslant (1+C_4)^p$ and finally

$$\lambda \leqslant \mu(1+C_4)^p. \quad \square$$

REMARK 1. Let $1 < p \leqslant 2$. We assume that Ω is a strictly convex smooth bounded domain in $\mathbb{R}^n$ with $n \geqslant 2$ such that $x \cdot \nu \geqslant k > 0$ with some $k > 0$ for all $x \in \partial\Omega$, where ν is the unit outer normal vector to $\partial\Omega$ at x. Then according to Proposition 2.3 in [20], we have the same statement as Proposition 1 in this paper. Hence also in this case, the conclusion of Theorem 3 holds.

REMARK 2. If the solution of (1.2) satisfies the result of Proposition 1, we can prove the quenching result for large $\lambda > 0$. However since the proof of the proposition is done by the maximum principle, it is impossible to lead us to the quenching result in this manner.

5. Discussion

In this section, we compare our results with those of [28, 29]. In [29], they consider the following damped MEMS equation with nonlocal term:

$$\begin{cases} u_{tt} + u_t = \Delta u + \dfrac{\lambda}{(1-u)^2\left(1+\int_\Omega \frac{dx}{1-u}\right)^2}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) < 1, & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{5.1}$$

where Ω is a bounded domain in $\mathbb{R}^n (n = 1,2,3)$ with smooth boundary $\partial\Omega$ and $T > 0$ is a maximal existing time of the solution u. Different from (1.7), we do not have to assume the symmetry of the solution u and domain Ω. Owing to the damping term u_t, the second order derivative estimates $\|u\|_{H^2(\Omega)}$, $\|u_t\|_{H^1(\Omega)}$, $\|u_{tt}\|_{L^2(\Omega)}$ are obtained. For instance, modifying (5.1) as constructed in Section 2 and multiplying this modified equation by $2u$, we have

$$\frac{d}{dt}\int_\Omega \left(u_t^2 + |\nabla u|^2\right)dx + 2\int_\Omega u_t^2 dx \leqslant \frac{8\lambda}{\delta^2}|\Omega|\,\|u_t\|_{L^2(\Omega)},$$

where δ is the constant defined in Section 2 and $|\Omega|$ is a measure of Ω in $\mathbb{R}^n$. Hence we can control $\|u_t\|_{L^2(\Omega)}$ in the right-hand side by the Cauchy-Schwartz inequality. Compared to (1.2), this estimate is the advantage of (5.1). Finally, they have the result of global existence of a solution for higher dimensional domain.

THEOREM 4. (Theorem 3.1 in [29]) *Suppose that* $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$, $0 \leqslant u_0(x) \leqslant 1-2\delta$ *for some* $\delta \in (0,1/2)$ *and* $v_0 \in H_0^1(\Omega)$. *There exist two constants* $\gamma > 0$ *and* $\hat{\lambda}$ *depending only on* Ω, n *and* δ *such that* (5.1) *has a unique global solution satisfying*

$$u \in C\big([0,\infty); H^2(\Omega) \cap H_0^1(\Omega)\big) \cap C^1\big([0,\infty); H_0^1(\Omega)\big) \cap C^2\big([0,\infty); L^2(\Omega)\big), \tag{5.2}$$

provided that $\|u_0\|_{H^2(\Omega)} + \|v_0\|_{H^1(\Omega)} \leqslant \gamma$ *and* $0 < \lambda \leqslant \hat{\lambda}$.

We note that (1.5) has a solution for $0 < \lambda < \lambda^*$ by [14]. Owing to the nonlocal term, we can not expect the monotonicity of the nonlinear terms of (1.2) and (5.1). Thus it is not clear whether $\hat{\lambda} = \lambda^*$ or not. Moreover the method of proof of Theorem 4 is also applicable to that of stability theorem. Then they show that the global solution converges to the stationary solution exponentially. For more details, see Corollary 3.1 in [29]. In [28], they consider the following damped MEMS equation without nonlocal term:

$$\begin{cases} \varepsilon u_{tt} + u_t = \Delta u + \frac{\lambda}{(1-u)^2}, & x \in \Omega,\ t \in (0,T), \\ u(x,t) = 0, & x \in \partial\Omega,\ t \in (0,T), \\ u(x,0) = u_0(x) < 1, & x \in \Omega, \\ u_t(x,0) = v_0(x), & x \in \Omega, \end{cases} \tag{5.3}$$

where $\varepsilon > 0$ and $\lambda > 0$. In the corresponding stationary problem (1.4), according to [6, 24], there exists a constant λ^* such that we have solutions w for $0 < \lambda < \lambda^*$ and no solution for $\lambda > \lambda^*$. We take advantage of comparison principle to show that (1.4) has a minimal solution w_λ for $0 < \lambda < \lambda^*$ with the positive eigenvalue. In (1.5), we can not derive the minimality of solution. Hence, in addition to Theorem 4, they prove $\hat{\lambda} = \lambda^*$ and the quenching result.

THEOREM 5. (Theorem 2.1 in [28]) *Assume that* $1 \leqslant n \leqslant 3$, $\varepsilon \in (0,1]$ *and* $0 < \lambda < \lambda^*$. *Suppose that* $u_0 \in H^2(\Omega) \cap H_0^1(\Omega)$ *and* $v_0 \in H_0^1(\Omega)$. *There exists a constant* $\gamma > 0$ *independent of* ε *such that if*

$$\|u_0 - w_\lambda\|_{H^2(\Omega)} + \|v_0\|_{H^1(\Omega)} + \sqrt{\varepsilon}\left\| \frac{\lambda}{(1-u_0)^2} + \Delta u_0 - v_0 \right\|_{L^2(\Omega)} \leqslant \gamma,$$

then (5.3) *has a unique global solution satisfying* (5.2). *Furthermore, the solution has the following asymptotic stability*:

$$\|u - w_\lambda\|_{H^2(\Omega)} + \|u_t\|_{H^1(\Omega)} + \sqrt{\varepsilon}\,\|u_{tt}\|_{L^2(\Omega)} \leqslant Ce^{-\alpha t},$$

where $\alpha > 0$ *and* $C > 0$ *are constants independent of* t *and* ε.

On the other hand, for $0 < \lambda < \lambda^*$, they show that the solution with sufficiently large initial data quenches in finite time by deriving the energy inequality. For more details, see Theorem 2.2 in [28]. By the Kaplan's argument [22] and the existence of minimal solution, they show that the solution must quench in finite time for any $\lambda > \lambda^*$. For more details, see Theorem 3.1 in [28]. From the viewpoint of physics, it is important to study the asymptotic behaviour of solution as $\varepsilon \to 0$ and to argue the relation between (5.3) and (1.6) with $\beta = 2$. This limit is said to be the viscosity dominated limit. If the initial functions are sufficiently smooth and satisfy the suitable compatibility conditions, they approximate the solution of (5.3) by that of (1.6) plus initial layers in the power of ε. For more details, see Theorem 4.1 in [28].

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