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CORRIGENDUM TO: POSITIVE SOLUTIONS FOR A FOURTH ORDER DIFFERENTIAL INCLUSION WITH BOUNDARY VALUES, PUBLISHED IN DIFFERENTIAL EQUATIONS AND APPLICATIONS VOL. 8 NO. 1 (2016), 21—31, BY JOHN S. SPRAKER

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Abstract. In Theorem 3 of [2] I included an extension to the Ascoli theorem. While the statement of the theorem and its later use were correct, the proof has a slight error which I noticed while in the process of writing a sequel. Also a few comments about the complete continuity of an operator are provided as well as well as an additional reference.

1. Introduction

In [2] a small part of the proof of Theorem 3 was in error, though the statement of the theorem is correct. We will correct the error below. In addition to this there was also an argument in [2] that a certain operator, A, was completely continuous. We include an additional reference to justify this complete continuity argument.

2. Main results

THEOREM 3 [2]. *Suppose $T \subset C_0^1[0,1]$ is closed and has the following properties:*

1) $\sup_{f \in T} \| f' \|_{C[0,1]} < \infty$*;*

2) for all $\varepsilon > 0$, and for all $t \in [0,1]$, there exists $\delta = \delta(t,\varepsilon)$ such that for any $y \in [0,1]$ with $|t-y| < \delta$ we have $|f'(t) - f'(y)| < \varepsilon$ (i.e. $\{f' | f \in T\}$ is equicontinuous).

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Then T is compact in $C_0^1[0,1]$.

Proof. We correct the error in the proof in [2]. Let $Y = \sup_{f\in T} \| f' \|_{C[0,1]}$. Let $\varepsilon > 0$ and $t, y \in [0,1]$.

$$|f(y)-f(t)| = \left|\int_0^y f'(x)dx - \int_0^t f'(x)dx\right| = \left|\int_t^y f'(x)dx\right| \leqslant \left|\int_t^y \left|f'(x)\right| dx\right|$$
$$\leqslant Y\left|\int_t^y dx\right| \leqslant (Y+1)\,|t-y|.$$

Now we can choose $\delta = \frac{\varepsilon}{Y+1}$ since $Y+1>0$. Clearly whenever $|t-y|<\delta$ we will have $|f(y)-f(t)| < (Y+1)\frac{\varepsilon}{Y+1} = \varepsilon$. Thus T is equicontinuous. (The error in [2] involved choosing a δ which depended on the choice of $f \in T$. Such a δ cannot, of course, be used to prove equicontinuity.) The rest of the proof in [2] is correct. □

In addition to this there was an argument in [2] that a certain operator, A, was completely continuous. The actual result comes from the following theorem which is a special case of Proposition 1.7 in [1].

THEOREM [1]. *Let $\Phi : [0,1]\times R^2 \to R$ be L^1-Caratheodory. Let E_1 and E_2 be Banach Spaces. Let $\varphi : C([0,1],R^2) \to L^1([0,1],R)$ be the mapping $\varphi(x) = \{\eta \in L^1([0,1],R) : \eta(t) \in \Phi(t,x(t))$ a.e on $[0,1]\}$ and let $T_1 : E_1 \to C([0,1],R^2)$ and $T_2 : L^1([0,1],R) \to E_2$ be continuous linear mappings. Assume further that for each bounded set $B \subseteq C([0,1],R^2)$ the set $\overline{T_2 \circ \varphi(B)}$ is compact. Then the multivalued mapping $P \equiv T_2 \circ \varphi \circ T_1 : E_1 \to E_2$ is completely continuous.*

In order to apply this theorem in our case we let $E_1 = C^1[0,1]\times C^1[0,1]$, $E_2 = C^1[0,1]$, and $\Phi = G$ where G is as specified in [2]. $T_1 : C^1[0,1]\times C^1[0,1] \to C([0,1],R^2)$ and $T_2 : L^1([0,1],R) \to C^1[0,1]$ will be defined by

$$T_1(f,g) = (f(t)-g(t), f'(t)-g'(t))$$

and

$$T_2h(t) \equiv \lambda \int_0^t \left(\int_0^\tau \left[\int_r^1 \left\{\int_s^1 h(\nu)d\nu\right\} ds\right] dr\right) d\tau,$$

respectively. In [2] the operator $A : C^1[0,1] \to C^1[0,1]$ was defined by $Av = \{w \in AC^3[0,1] | w(t) = T_2h(t)$ where $h(t) \in G(t, v(t)-z(t), v'(t)-z'(t))$ $a.e$ on $[0,1]\}$. $z(t)$ was a particular function. Observe that A is just the restriction of P to the set $\underline{C^1[0,1]\times}$ $\{z\}$. Clearly A will be completely continuous if P is. In [2] it was shown that $\overline{A(B)}$ was compact for a set B which was bounded in $C_0^1[0,1]$. Actually, in order to apply the above theorem we need to work with $\overline{T_2 \circ \varphi(B)}$ and sets that are bounded in $C([0,1],R^2)$. That does not really alter the proof found in [2], because the compactness result still follows from the integral boundedness condition and Theorem 3.

REFERENCES

[1] T. PRUSZKO, *Topological degree methods in multivalued boundary value problems*, J. NONLINEAR ANAL., **5** (1981), 953–973.

[2] J. S. SPRAKER, *Positive solutions for a fourth order differential inclusion with boundary values*, DIFFER. EQU. APPL., **8** (2016), NO. 1, 21–31.

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