Journal of Classical Analysis
Volume 3, Number 1 (2013), 59–67
doi:10.7153/jca-03-05

NEW MATRIX FORMULAS FOR LAGUERRE MATRIX POLYNOMIALS

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Abstract. In this paper, we obtain some properties for Laguerre matrix polynomials. The relations between Laguerre and Jacobi matrix polynomials in this study are indicated. We also derive multilinear and multilateral generating matrix functions for Laguerre matrix polynomials.

1. Introduction

In the recent papers, matrix polynomials have significant emergent. Some results in the theory of classical orthogonal polynomials have been extended to orthogonal matrix polynomials, see [1, 2, 3, 4, 6]. In [5], these matrix polynomials are orthogonal as examples of right orthogonal matrix polynomial sequences for appropriate right matrix moment functionals of integral type. Jacobi and Laguerre matrix polynomials have been introduced and studied in [1, 6] for matrices in $\mathbb{C}^{N\times N}$. Our main aim in this paper is to prove new properties for the Laguerre matrix polynomials. The outline of this paper is as follows. In section 2, we demonstrate some properties of the Laguerre matrix polynomials. The relations between Laguerre and Jacobi matrix polynomials in this study are also indicated in section 3. Multilinear and multilateral generating matrix functions for Laguerre matrix polynomials are derived in section 4.

Throughout this paper, for a matrix A in $\mathbb{C}^{N\times N}$, its spectrum $\sigma(A)$ denotes the set of all eigenvalues of A. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane and A is a matrix in $\mathbb{C}^{N\times N}$ with $\sigma(A)\subset\Omega$, then from the properties of the matrix functional calculus in [7], it follows that: $f(A)g(A)=g(A)f(A)$. Hence, if $B\in\mathbb{C}^{N\times N}$ is a matrix for which $\sigma(B)\subset\Omega$ and if $AB=BA$, then $f(A)g(B)=g(B)f(A)$. We say that the matrix A in $\mathbb{C}^{N\times N}$ is a positive stable if $Re(\lambda)>0$ for all $\lambda\in\sigma(A)$. Furthermore the identity matrix of $\mathbb{C}^{N\times N}$ will be denoted by I.

LEMMA 1. *For matrices $A(k,n)$ and $B(k,n)$ in $\mathbb{C}^{N\times N}$ where $n\geqslant 0,\ k\geqslant 0$ the following relations are satisfied by Defez and Jódar in* [2]*:*

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{n}A(k,n-k) \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}A(k,n)=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}A(k,n+k). \tag{1.2}$$

Mathematics subject classification (2010): 33C45, 15A60.
Keywords and phrases: Laguerre matrix polynomials, Jacobi matrix polynomials, generating matrix function, beta matrix function.

From [3], one can see

$$(P)_n = P(P+I)(P+2I)\ldots(P+(n-1)I);\ n \geqslant 1;\ (P)_0 = I.$$

The hypergeometric matrix function ${}_2F_1(A,B;C;z)$ has been given in [3]

$${}_2F_1(A,B;C;z) = \sum_{n=0}^{\infty} \frac{(A)_n (B)_n}{n!} \left[(C)_n\right]^{-1} z^n$$

for matrices A,B and C in $\mathbb{C}^{N\times N}$ such that $C+nI$ is invertible for all integer $n \geqslant 0$ and for $|z| < 1$.

Let A be a matrix in $\mathbb{C}^{N\times N}$ where $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha > 0$ and λ be a complex number whose real part is positive. Then the Laguerre matrix polynomials $L_n^{(A,\lambda)}(x)$ are defined by [6]:

$$L_n^{(A,\lambda)}(x) = \sum_{k=0}^{n} \frac{(-1)^k}{k!(n-k)!} (A+I)_n \left[(A+I)_k\right]^{-1} (\lambda x)^k \ ;\ \ n \geqslant 0. \tag{1.3}$$

Such matrix polynomials have following generating matrix function:

$$\sum_{n=0}^{\infty} L_n^{(A,\lambda)}(x) t^n = (1-t)^{-A-I} e^{-\frac{\lambda x t}{1-t}}\ ;\ x \in \mathbb{C},\ t \in \mathbb{C},\ |t| < 1. \tag{1.4}$$

The Jacobi matrix polynomials have been given in [1], $P_n^{(A,B)}(x)$ for parameter matrices A and B whose eigenvalues, z, all satisfy $Re(z) > -1$. For any natural number $n > 0$, the Jacobi matrix polynomials $P_n^{(A,B)}(x)$ are defined by

$$P_n^{(A,B)}(x) = \frac{(-1)^n}{n!}\, {}_2F_1\left(A+B+nI, -nI; B+I; \frac{1+x}{2}\right) \Gamma^{-1}(B+I)\Gamma(B+(n+1)I) \tag{1.5}$$

or

$$P_n^{(A,B)}(x) = \frac{1}{n!}\, {}_2F_1\left(A+B+nI, -nI; A+I; \frac{1-x}{2}\right) \Gamma^{-1}(A+I)\Gamma(A+(n+1)I). \tag{1.6}$$

DEFINITION 1. Let P be a positive stable matrix in $\mathbb{C}^{N\times N}$, then Gamma matrix function in [4] is defined by

$$\Gamma(P) = \int_0^{\infty} e^{-t}\, t^{P-I} dt\ , \quad t^{P-I} = \exp\left[(P-I)\ln t\right]. \tag{1.7}$$

DEFINITION 2. Let P and Q be positive stable matrices in $\mathbb{C}^{N\times N}$, then Beta matrix function in [4] is defined by

$$\mathscr{B}(P,Q) = \int_0^{1} t^{P-I}\ (1-t)^{Q-I}\ dt. \tag{1.8}$$

LEMMA 2. *Let P,Q, $P+Q$ be positive stable matrices in $\mathbb{C}^{N\times N}$ and $PQ=QP$, then*

$$\mathscr{B}(P,Q)=\Gamma(P)\Gamma(Q)\Gamma^{-1}(P+Q) \tag{1.9}$$

[4].

2. Some results for Laguerre matrix polynomials

THEOREM 1. *Let A and B be matrices in $\mathbb{C}^{N\times N}$ satisfying spectral condition $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha>0$, $(-\beta)$ is not an eigenvalue of B for every integer $\beta>0$. Then Laguerre matrix polynomials yield following equation:*

$$\sum_{k=0}^{n}\frac{(A-B)_{n-k}L_k^{(B,\lambda)}(x)}{(n-k)!}=L_n^{(A,\lambda)}(x) \tag{2.1}$$

where $AB=BA$, $n\geqslant 0$ and $\operatorname{Re}(\lambda)>0$.

Proof. Using (1.2), we can write

$$\begin{aligned}&\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A-B)_{n-k}L_k^{(B,\lambda)}(x)}{(n-k)!}t^n\\ &=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(A-B)_{n}L_k^{(B,\lambda)}(x)}{n!}t^{n+k}\\ &=\left(\sum_{n=0}^{\infty}\frac{(A-B)_n}{n!}t^n\right)\left(\sum_{k=0}^{\infty}L_k^{(B,\lambda)}(x)t^k\right).\end{aligned} \tag{2.2}$$

On the other hand, we have

$$\sum_{n=0}^{\infty}\frac{(A-B)_n}{n!}t^n=(1-t)^{-(A-B)},\quad |t|<1. \tag{2.3}$$

Using (2.3) and (1.4) in (2.2), we get

$$\begin{aligned}\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A-B)_{n-k}L_k^{(B,\lambda)}(x)}{(n-k)!}t^n&=(1-t)^{-(A-B)}(1-t)^{-B-I}e^{-\frac{\lambda xt}{1-t}}\\ &=(1-t)^{-A-I}e^{-\frac{\lambda xt}{1-t}}\\ &=\sum_{n=0}^{\infty}L_n^{(A,\lambda)}(x)t^n.\end{aligned}$$

Comparing coefficients of t^n in last equation, we have desired relation. □

THEOREM 2. *Let A be a matrix in $\mathbb{C}^{N\times N}$ where $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha>0$ and $\operatorname{Re}(\lambda)>0$. Then Laguerre matrix polynomials satisfy:*

$$\sum_{k=0}^{n}\frac{(A+(k+1)I)_{n-k}L_k^{(A,\lambda)}(z)}{(n-k)!}w^k(1-w)^{n-k}=L_n^{(A,\lambda)}(zw);\quad |w|<1. \tag{2.4}$$

Proof. Starting from left-hand side of (2.4) and using (1.2), we have

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A+(k+1)I)_{n-k}L_k^{(A,\lambda)}(z)}{(n-k)!}w^k(1-w)^{n-k}t^n$$
$$=\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{(A+(k+1)I)_{n}L_k^{(A,\lambda)}(z)}{n!}w^k(1-w)^{n}t^{n+k}$$
$$=\sum_{k=0}^{\infty}L_k^{(A,\lambda)}(z)\left(\sum_{n=0}^{\infty}\frac{(A+(k+1)I)_{n}}{n!}(1-w)^{n}t^{n}\right)(wt)^k.$$

On the other hand, we get

$$\sum_{n=0}^{\infty}\frac{(A+(k+1)I)_{n}}{n!}(1-w)^{n}t^{n}=(1-t+tw)^{-(A+(k+1)I)};\ \ |t-tw|<1. \tag{2.5}$$

By using (1.4) and (2.5), we obtain

$$\sum_{n=0}^{\infty}\sum_{k=0}^{n}\frac{(A+(k+1)I)_{n-k}L_k^{(A,\lambda)}(z)}{(n-k)!}w^k(1-w)^{n-k}t^n$$
$$=\sum_{k=0}^{\infty}L_k^{(A,\lambda)}(z)\left((1-t+tw)^{-(A+(k+1)I)}\right)(wt)^k$$
$$=(1-t+tw)^{-A-I}\sum_{k=0}^{\infty}L_k^{(A,\lambda)}(z)\left(\frac{wt}{1-t+tw}\right)^k;\ \ \left|\frac{wt}{1-t+tw}\right|<1$$
$$=(1-t+tw)^{-A-I}\left(1-\frac{wt}{1-t+tw}\right)^{-A-I}e^{-\frac{\lambda z\left(\frac{wt}{1-t+tw}\right)}{1-\left(\frac{wt}{1-t+tw}\right)}}$$
$$=(1-t)^{-A-I}e^{-\frac{\lambda zwt}{1-t}}$$
$$=\sum_{n=0}^{\infty}L_n^{(A,\lambda)}(zw)t^n.$$

Comparing of the coefficients of t^n, theorem is proved. □

THEOREM 3. *Let A and C be matrices in $\mathbb{C}^{N\times N}$ where $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha>0$ and* $\operatorname{Re}(\lambda)>0$. *Then Laguerre matrix polynomials satisfy:*

$$\sum_{n=0}^{\infty}(C)_n(A+I)_n^{-1}L_n^{(A,\lambda)}(z)w^n=(1-w)^{-C}\ {}_1F_1\left(C;A+I;\frac{-\lambda zw}{1-w}\right);\ \ |w|<1 \tag{2.6}$$

where $AC=CA$.

Proof. Starting from right-hand side of (2.6), we can write

$$(1-w)^{-C}\ {}_1F_1\left(C;A+I;\frac{-\lambda zw}{1-w}\right)=\sum_{k=0}^{\infty}\frac{(C)_k(A+I)_k^{-1}}{k!}(-\lambda zw)^k(1-w)^{-C-kI}.$$

In the last equation, if we consider Taylor expansion of $F(w) = (1-w)^{-C-kI}$ at $w=0$, we can write $F(w) = \sum_{n\geqslant 0} \frac{(C)_{n+k}}{n!}\left[(C)_k\right]^{-1} w^n$. Then using (1.1) and (1.3), theorem can be proved. □

THEOREM 4. *Let A be a matrix in $\mathbb{C}^{N\times N}$ where $(-\alpha)$ is not an eigenvalue of A for every integer $\alpha > 0$ and $\operatorname{Re}(\lambda) > 0$. Then Laguerre matrix polynomials satisfy:*

$$\sum_{k=0}^{\infty} \frac{L_n^{(A+kI,\lambda)}(z)}{k!} w^k = e^w L_n^{(A,\lambda)}(\lambda z - w). \tag{2.7}$$

Proof. Starting from left-hand side of (2.7) and using (1.4), we get

$$\begin{aligned}
\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} \frac{L_n^{(A+kI,\lambda)}(z)}{k!} w^k t^n &= \sum_{k=0}^{\infty}\left\{\sum_{n=0}^{\infty} L_n^{(A+kI,\lambda)}(z) t^n\right\}\frac{w^k}{k!} \\
&= \sum_{k=0}^{\infty}\left\{(1-t)^{-A-kI-I} e^{-\frac{\lambda z t}{1-t}}\right\}\frac{w^k}{k!} \\
&= (1-t)^{-A-I} e^{-\frac{\lambda z t}{1-t}} \sum_{k=0}^{\infty} \frac{\left(\frac{w}{1-t}\right)^k}{k!} \\
&= (1-t)^{-A-I} e^{-\frac{\lambda z t}{1-t}} e^{\frac{w}{1-t}} \\
&= e^w (1-t)^{-A-I} e^{-\frac{t(\lambda z - w)}{1-t}}.
\end{aligned}$$

Also again using (1.4) and comparing of the coefficients of t^n, we get proof of theorem. □

THEOREM 5. *Let A and B be matrices in $\mathbb{C}^{N\times N}$ satisfying spectral condition $\operatorname{Re}(z) > -1$ for every eigenvalue $z \in \sigma(A)$, $\operatorname{Re}(z) > -1$ for every eigenvalue $z \in \sigma(B)$ and $(A-B)$ be a positive stable matrix in $\mathbb{C}^{N\times N}$. Then Laguerre matrix polynomials yield following equation:*

$$\begin{aligned}
L_{m+n}^{(A,\lambda)}(x)\left[L_{m+n}^{(A,\lambda)}(0)\right]^{-1} &= \Gamma(A+I)\Gamma^{-1}(A-B)\Gamma(B+I)\times \\
\int_0^1 t^B (1-t)^{A-B-I} L_m^{(B,\lambda)}(xt)&\left[L_m^{(B,\lambda)}(0)\right]^{-1} L_n^{A-B-I}(x(1-t))\left[L_n^{A-B-I}(0)\right]^{-1} dt
\end{aligned} \tag{2.8}$$

where $AB = BA$, $n,m \geqslant 0$ and $\operatorname{Re}(\lambda) > 0$.

Proof. By using (1.3), right-hand side of equation in (2.8) can be written

$$\begin{aligned}
S &= \Gamma(A+I)\Gamma^{-1}(A-B)\Gamma(B+I) \\
&\quad \times \int_0^1 t^B (1-t)^{A-B-I} L_m^{(B,\lambda)}(xt)\left[L_m^{(B,\lambda)}(0)\right]^{-1} L_n^{A-B-I}(x(1-t))\left[L_n^{A-B-I}(0)\right]^{-1} dt
\end{aligned}$$

$$= \Gamma(A+I)\Gamma^{-1}(A-B)\Gamma(B+I)$$
$$\times \int_0^1 t^B (1-t)^{A-B-I} \left(\sum_{k=0}^{\infty} \frac{(-mI)_k}{k!} (B+I)_m \left[(B+I)_k\right]^{-1} (t\lambda x)^k \right)$$
$$\times (B+I)_m^{-1} \left(\sum_{l=0}^{\infty} \frac{(-nI)_l}{l!} (A-B)_n \left[(A-B)_l\right]^{-1} \left[(1-t)\lambda x\right]^l \right) \left[(A-B)_n\right]^{-1} dt.$$

By making necessary arrangements and using Beta matrix function in (1.8) and equation in (1.9), we obtain

$$S = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(-m)_k}{k!} \frac{(-n)_l}{l!} \left[(A+I)_{k+l}\right]^{-1} (\lambda x)^{k+l}.$$

By using (1.1) and following equation

$$\sum_{l=0}^{k} \frac{(-n)_l (-m)_{k-l}}{l!(k-l)!} = \frac{(-m-n)_k}{k!},$$

we get proof of theorem. □

3. Connections between Laguerre and Jacobi matrix polynomials

THEOREM 6. *Let A and B be matrices in $\mathbb{C}^{N\times N}$ satisfying spectral condition* $\operatorname{Re}(z) > -1$ *for every eigenvalue* $z \in \sigma(A)$, $\operatorname{Re}(z) > -1$ *for every eigenvalue* $z \in \sigma(B)$, $\operatorname{Re}(z) > 0$ *for every eigenvalue* $z \in \sigma(A+B)$ *and* $\operatorname{Re}(\lambda) > 0$. *Then connection between Laguerre and Jacobi matrix polynomials is*

$$P_n^{(A,B)}(x) = \Gamma^{-1}(A+B+nI+I) \int_0^{\infty} t^{A+B+nI} e^{-t} L_n^{(A,\lambda)}\left(\frac{(1-x)t}{2\lambda}\right) dt. \tag{3.1}$$

Proof. By using (1.3), right-hand side of equation in (3.1) can be written

$$\Gamma^{-1}(A+B+nI+I) \int_0^{\infty} t^{A+B+nI} e^{-t} L_n^{(A,\lambda)}\left(\frac{(1-x)t}{2\lambda}\right) dt$$
$$= \Gamma^{-1}(A+B+nI+I) \sum_{k=0}^{n} \left\{ \frac{(-nI)_k}{k!\, n!} \left(\frac{1-x}{2}\right)^k \right.$$
$$\left. \times \left\{ \int_0^{\infty} t^{A+B+nI+kI} e^{-t} dt \right\} (A+I)_n \left[(A+I)_k\right]^{-1} \right\}.$$

Also using Gamma matrix function to evaluate integral and making necessary arrangements, we obtain the required relation. □

THEOREM 7. *Let* A *be a matrix in* $\mathbb{C}^{N\times N}$ *satisfying spectral condition* $\operatorname{Re}(z) > -1$ *for every eigenvalue* $z\in\sigma(A)$ *and* $\operatorname{Re}(\lambda)>0$. *Then connection between Laguerre and Jacobi matrix polynomials is*

$$\lim_{s\to\infty} P_n^{(A,Is)}(1-2x\lambda s^{-1}) = L_n^{(A,\lambda)}(x) \tag{3.2}$$

where $s\in(0,\infty)$.

Proof. Using (1.6) in left-hand side of (3.2), theorem can be proved. □

4. Multilinear and multilateral generating matrix functions for the Laguerre matrix polynomials

In this section, we derive several families of bilinear and bilateral generating matrix functions for the Laguerre matrix polynomials generated by (1.4). We first state our result as the following.

THEOREM 8. *Corresponding to a non-vanishing function* $\Omega_\mu(y_1,\ldots,y_s)$ *of* s *complex variables* $y_1,\ldots,y_s$ $(s\in\mathbb{N})$ *and of complex order* μ, *let*

$$\Lambda_{\mu,\nu}(y_1,\ldots,y_s;z) := \sum_{k=0}^{\infty} a_k\Omega_{\mu+\nu k}(y_1,\ldots,y_s)z^k;\ \ (a_k\neq 0,\ \mu,\nu\in\mathbb{C}) \tag{4.1}$$

and

$$\Theta_{n,p,\mu,\nu}(x;y_1,\ldots,y_s;\zeta) := \sum_{k=0}^{[n/p]} a_k\, L_{n-pk}^{(A,\lambda)}(x)\, \Omega_{\mu+\nu k}(y_1,\ldots,y_s)\zeta^k \tag{4.2}$$

where A *is a matrix in* $\mathbb{C}^{N\times N}$ *whose eigenvalues,* z, $(-z)$ *is not an eigenvalue of* A *for every integer* $z>0$, $n,p\in\mathbb{N}$ *and (as usual)* $[\alpha]$ *represents the greatest integer in* $\alpha\in\mathbb{R}$. *Then we have*

$$\sum_{n=0}^{\infty}\Theta_{n,p,\mu,\nu}\left(x;y_1,\ldots,y_s;\frac{\eta}{t^p}\right)t^n = (1-t)^{-A-I}e^{-\frac{\lambda xt}{1-t}}\,\Lambda_{\mu,\nu}(y_1,\ldots,y_s;\eta) \tag{4.3}$$

provided that each member of (4.3) *exists for* $|t|<1$ *and* $\operatorname{Re}(\lambda)>0$.

Proof. For convenience, let S denote the first member of the assertion (4.3) of Theorem 8. Then, plugging the polynomials

$$\Theta_{n,p,\mu,\nu}\left(x;y_1,\ldots,y_s;\frac{\eta}{t^p}\right)$$

which comes from (4.2) into the left-hand side of (4.3), we obtain

$$S=\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} a_k\, L_{n-pk}^{(A,\lambda)}(x)\,\Omega_{\mu+\nu k}(y_1,\ldots,y_s)\,\eta^k\, t^{n-pk}. \tag{4.4}$$

Upon changing the order of summation in (4.4), if we replace n by $n+pk$, we can write

$$\begin{aligned} S &= \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} a_k L_n^{(A,\lambda)}(x)\, \Omega_{\mu+\nu k}(y_1,\ldots,y_s)\, \eta^k\, t^n \\ &= \left(\sum_{n=0}^{\infty} L_n^{(A,\lambda)}(x)\, t^n\right)\left(\sum_{k=0}^{\infty} a_k\, \Omega_{\mu+\nu k}(y_1,\ldots,y_s)\, \eta^k\right) \\ &= (1-t)^{-A-I} e^{-\frac{\lambda xt}{1-t}} \Lambda_{\mu,\nu}(y_1,\ldots,y_s;\eta), \end{aligned}$$

which completes the proof of Theorem 8. □

If we set $s=2$ and $\Omega_{\mu+\nu k}(y,z) = H_{\mu+\nu k}(y,z,B)$ in theorem 8, where the two-variable Hermite matrix polynomials $H_n(y,z,B)$ are defined by means of the generating matrix function in [8]

$$\exp\left(yt\sqrt{2B} - zt^2 I\right) = \sum_{n=0}^{\infty} \frac{1}{n!} H_n(y,z,B) t^n; \quad |t| < \infty \tag{4.5}$$

where B is a positive stable matrix in $\mathbb{C}^{N\times N}$ then we obtain the following result which provides a class of multilateral generating matrix functions for the two-variable Hermite and the Laguerre matrix polynomials.

EXAMPLE 1. Taking $a_k = \frac{1}{k!}$, $\mu = 0$, $\nu = 1$ and $|t| < 1$, we have

$$\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} L_{n-pk}^{(A,\lambda)}(x)\, \frac{H_k(y,z,B)}{k!} \eta^k t^{n-pk} = (1-t)^{-A-I} e^{-\frac{\lambda xt}{1-t}} \exp\left(y\eta\sqrt{2B} - z\eta^2 I\right)$$

where $|\eta| < \infty$.

EXAMPLE 2. Taking $a_k = 1$, $\mu = 0$, $\nu = 1$, $|t| < 1$, B is a matrix in $\mathbb{C}^{N\times N}$ whose eigenvalues, z, $(-z)$ is not an eigenvalue of B for every integer $z > 0$, $\mathrm{Re}(\lambda_1) > 0$ and $\mathrm{Re}(\lambda_2) > 0$, we have bilinear generating function for Laguerre matrix polynomials

$$\sum_{n=0}^{\infty}\sum_{k=0}^{[n/p]} L_{n-pk}^{(A,\lambda_1)}(x)\, L_k^{(B,\lambda_2)}(y)\, \eta^k t^{n-pk} = (1-t)^{-A-I} e^{-\frac{\lambda_1 xt}{1-t}}\, (1-\eta)^{-B-I} e^{-\frac{\lambda_2 y\eta}{1-\eta}}$$

where $|\eta| < 1$.

Furthermore, for every suitable choice of the coefficients a_k $(k \in \mathbb{N}_0)$, if the multivariable function $\Omega_{\mu+\psi k}(y_1,\ldots,y_s)$, $(s \in \mathbb{N})$, is expressed as an appropriate product of several simpler functions, the assertions of theorem 8 can be applied in order to derive various families of multilinear and multilateral generating matrix functions for the Laguerre matrix polynomials.

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(Received March 4, 2013)

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