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ON THE DEGREE OF APPROXIMATION OF SIGNALS $\mathrm{Lip}(\alpha,r)$, $(r \geqslant 1)$ CLASS BY ALMOST RIESZ MEANS OF ITS FOURIER SERIES

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Abstract. In the present paper, an attempt is made to obtain the degree of approximation of signals (functions) belonging to the $\mathrm{Lip}(\alpha,r)$ class, using almost Riesz summability method of its Fourier series, so that some theorems become particular case of our main theorem. Analysis of signals or time functions is of great importance, because it conveys information or attributes of some phenomenon. The engineers and scientists use properties of Fourier approximation for designing digital filters.

1. Introduction

Broadly speaking, signals are treated as functions of one variable and images are represented by functions of two variables. The study of these concepts is directly related to the emerging area of information technology. Especially, Psarakis and Moustakides [19] presented a new L_2 based method for designing the Finite Impulse Response (FIR) digital filters and get corresponding optimum approximations having improved performance. Mittal and Rhoades [11, 13] and Mittal et al. [14] have initiated the studies of error estimates $\mathrm{E}_n(\mathrm{f})$ through Trigonometric Fourier Approximation (TFA) for the situations in which the summability matrix T does not have monotone rows. Mittal and Mishra [9] have determined the approximation of signals (functions) belonging to the weighted class by almost matrix summability means of its Fourier series. Several authors including Lorentz [5], King [4] and Nanda [15] have studied almost convergent sequences. Just as convergence gives rise to absolute and strong convergence, it was quite natural to expect that almost convergence must give rise to the concepts of absolute almost and strong almost convergence. Absolutely almost and strongly almost convergent sequences have been introduced and discussed in a natural way by Das, Kuttner and Nanda [1] and Maddox [6] respectively.

The summability methods of real or complex sequences by infinite matrices are of three types (see Maddox [7], p. 185) – ordinary, absolute and strong. In the same vein, it is expected that the concept of almost convergence must give rise to three types of summability methods – almost, absolutely almost and strongly almost. The almost summable sequences have been discussed by King [4], Schaefer [19] and some others.

The degree of approximation of functions f belonging to the classes $\mathrm{Lip}\,\alpha$, $\mathrm{Lip}(\alpha,r)$, $\mathrm{Lip}(\xi(t),r)$ and $W(L^r,\xi(t))$ (Khan[2]) by Nörlund (N_p) and matrix summability

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methods has been determined by various investigators such as Khan([2], [3]), Mittal & Mishra [9]. The approximation of signals using almost Riesz means has not been studied so far. In this paper, a new theorem on the degree of approximation of signal $f \in \mathrm{Lip}(\alpha, r)$, $(r \geqslant 1)$ by almost Riesz means of its infinite Fourier series has been established.

2. Definitions and notations

Let S be the set of all real or complex sequences and let l_∞, c and c_0 denote the Banach spaces of bounded, convergent and null sequences $x = \{x_k\}_{k=0}^{\infty}$ respectively, normed as usual by $\|x\| = \sup_{k \geqslant 0} |x_k|$. Let D be the shift operator on S, that is, $Dx = \{x_k\}_{k=1}^{\infty}$, $D^2 x = \{x_k\}_{k=2}^{\infty}$ and so on. It may be recalled that the Banach limit L is a non-negative linear functional on l_∞ such that L is invariant under the shift operator (that is, $L(D(x)) = L(x)$ for all $x \in l_\infty$) and that $L(e) = l$ where $e = \{1, 1, \ldots\}$. A sequence $x \in l_\infty$ is called almost convergent (see Lorentz [5]) if all Banach limits of x coincide. Let $\widehat{c}$ denote the set of all almost convergent sequences.

Lorentz [5] proved that

$$\widehat{c} = \left\{ x : \lim_{m \to \infty} \frac{1}{m+1} \sum_{i=0}^{m} x_{n+i} \text{ exists uniformly in } n \right\}.$$

Let f be a 2π-periodic signal (function) and $f \in L_1[0, 2\pi] = L_1$ (Lebesgue integrable), then the Fourier series of a function (signal) f at any point x is given by

$$f(x) \approx \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos kx + b_k \sin kx) \equiv \frac{a_0}{2} + \sum_{k=1}^{\infty} u_k(f; x), \tag{1}$$

with partial sums $s_n(f; x)$-a trigonometric polynomial of degree (or order) n, of the first $(n+1)$ terms.

A signal $f \in \mathrm{Lip}\,\alpha$, if $|f(x+t) - f(x)| = O(|t|^{\alpha})$ for $0 < \alpha \leqslant 1$.

A signal $f \in \mathrm{Lip}(\alpha, r)$, for $a \leqslant x \leqslant b$ if

$$w_r(t; f) = \left\{ \int_a^b |f(x+t) - f(x)|^r \, dx \right\}^{1/r} \leqslant M(|t|^{\alpha}), \quad r \geqslant 1, 0 < \alpha \leqslant 1 \tag{2}$$

where M is some absolute positive constant, not necessarily the same at each occurrence (see Def. 5.38 of McFadden [8].

If $r \to \infty$ in $\mathrm{Lip}(\alpha, r)$ class then this class reduces to $\mathrm{Lip}\,\alpha$.

Let $\sum a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let $\{p_n\}$ be a non-negative, non-decreasing, generating sequence of constants, real or complex for the (R, p_n) or R_p-method such that

$$P_n = \sum_{\gamma=0}^{n} p_\gamma \neq 0 \quad \forall n \geqslant 0, \quad P_{-1} = p_{-1} = 0 \text{ and } P_n \to \infty \text{ as } n \to \infty.$$

The sequence to sequence transformation $t_n = \sum_{k=0}^{n} (p_k/P_n)\, s_k$ defines the sequence $\{t_n\}$ of Riesz (R, p_n) means of the sequence $\{s_n\}$, generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be Riesz summable (R, p_n) to the sum s if $\lim_{n\to\infty} t_n$ exists and is equal to s, and is said to be absolutely (R, p_n) summable, or summable $|R, p_n|$, if the sequence $\{t_n\}$ is of bounded variation [symbolically $\{t_n\} \in BV$. Similarly by "$f(x) \in BV(a,b)$", we mean that $f(x)$ is a function of bounded variation in the interval (a,b) and by $\{\mu_n\} \in B$, where $\{\mu_n\}$ is a bounded sequence], that is $\sum_n |t_n - t_{n-1}| \leqslant K$. In the special case in which $p_n = 1$, $\forall n$ the Riesz R_p-mean reduces to the familiar Cesàro $(C,1)$-mean. The summability $|R, p_n|$, where $p_n = 1$, $\forall n$, is same as summability $|C,1|$.

According to Lorentz [5] a bounded sequence $\{s_k\}$ of k-th partial sums of an infinite Fourier series (1) is said to be almost convergent to s, if

$$\lim_{n\to\infty} s_{n,r} = \lim_{n\to\infty} \frac{s_r + s_{r+1} + \ldots + s_{r+n}}{n+1} = \lim_{n\to\infty} \frac{1}{(n+1)} \sum_{k=r}^{n+r} s_k = s, \tag{3}$$

Sharma and Qureshi [20] have defined that an infinite series $\sum u_n$ with the sequence of partial sums $\{s_n\}$ of infinite Fourier series (1) is called almost Riesz summable to a finite number s, provided

$$\tau_{n,r} = \frac{1}{P_n} \sum_{k=0}^{n} p_k s_{k,r} \to s, \text{ as } n \to \infty$$

uniformly with respect to r, where

$$s_{k,r} = \frac{1}{k+1} \sum_{\gamma=r}^{r+k} s_\gamma.$$

It is easy to see that a convergent sequence is almost convergent and the limits are the same. A bounded sequence $\{s_k\}$is said to be almost Riesz-summable to s if the Riesz-transform of $\{s_k\}$is almost convergent to s (see King [4]).

The Riesz mean (R, p_n) is regular if and only if $P_n \to \infty$ as $n \to \infty$ ([16], Theorem 1.4.4).

The L_r-norm is defined by

$$\|f\|_r = \left(\int_0^{2\pi} |f(x)|^r \, dx \right)^{1/r}, \quad r \geqslant 1.$$

The L_∞-norm of a function $f : R \to R$ is defined by

$$\|f\|_\infty = \sup\{|f(x)| : x \in R\}.$$

A signal (function) f is approximated by trigonometric polynomials $\tau_n(f;x)$ of order (or degree) n and the degree of approximation $E_n(f)$ of a function $f \in L_r$, $(f : R \to R)$ is given by

$$E_n(f) = \min_n \|f(x) - \tau_n(f;x)\|_r,$$

in terms of n.

The degree of approximation of a function $f : R \to R$ by a trigonometric polynomial $\tau_n(f;x)$ of order n under sup norm $\| \ \|_\infty$ is defined by [21]

$$\|\tau_n(f;x) - f(x)\|_\infty = \sup\{|\tau_n(f;x) - f(x)| : x \in R\}.$$

This method of approximation is called Trigonometric Fourier Approximation (TFA).

We write throughout the paper

$$\psi(t) = f(x+t) - f(x-t) - 2f(x),$$

$p(y) = p_{[y]}$ and $P(y) = P_{[y]}, P_\tau = P(1/t)$, where $\tau = [1/t]$ denotes the greatest integer not greater than $1/t$.

Throughout this paper K denotes a positive constant and it may be different in different relations.

3. Main theorem

It is well known that the theory of approximations i.e., TFA, which originated from a well known theorem of Weierstrass, has become an exciting interdisciplinary field of study for the last 130 years. Broadly speaking, signals are treated as functions of one variable and images are represented by functions of two variables. The study of these concepts is directly related to the emerging area of information technology. These approximations have assumed important new dimensions due to their wide applications in signal analysis [17], Speech Signal Analysis, Digital Communication in general, theory of Machines in Mechanical Engineering and in digital signal processing [18] in particular, in view of the classical Shannon sampling theorem.

This has motivated Khan ([2], [3]), Mittal, Rhoades ([10-12], [14]), Mittal et al. [13] and Mittal, Mishra [9] to obtain many results on TFA using summability methods without monotone rows of the matrix T: a digital filter.

The purpose of this paper is to determine the degree of approximation of signal $f \in \mathrm{Lip}(\alpha, r)$, $(r \geqslant 1)$ by almost Riesz summability means of its infinite Fourier series. We prove:

THEOREM 1. *If $f : R \to R$ is a 2π-periodic, Lebesgue integrable and belonging to the* $\mathrm{Lip}(\alpha, r)$, $(r \geqslant 1)$*-class, then the degree of approximation of a periodic function $f(x) \in \mathrm{Lip}(\alpha, r)$, $(r \geqslant 1)$ by almost Riesz means of its infinite Fourier series is given by*

$$\|\tau_{n,r}(f(t);x) - f(t)\|_r = O\left(P_n^{-\alpha+1/r}\right), \quad \forall n > 0, \tag{4}$$

and $\psi(t)$ satisfies the following conditions

$$\left\{\int_0^{\pi/P_n} \left(\frac{t\,|\psi(t)|}{t^\alpha}\right)^r dt\right\}^{1/r} = O\left(P_n^{-1}\right), \tag{5}$$

$$\left\{\int_{\pi/P_n}^{\pi} \left(\frac{t^{-\delta}\,|\psi(t)|}{t^\alpha}\right)^r dt\right\}^{1/r} = O\left(P_n^{\delta}\right), \tag{6}$$

where δ *is a finite quantity, Riesz means are regular and* $r^{-1}+s^{-1}=1$ *such that* $1 \leqslant r \leqslant \infty$.

4. Proof of Main Theorem 1

It is well known that the partial sums $s_{k,r}(f(t),x)$ of the Fourier series (1) is given by

$$s_{k,r}(f(t),x)-f(t) = \frac{1}{2\pi(k+1)}\int_0^{\pi} \psi(t)\frac{\cos(rt)-\cos(k+r+1)t}{2\sin^2\frac{t}{2}}dt.$$

Denoting almost Riesz mean of partial sums $s_{k,r}(f(t),x)$ by $\tau_{n,r}(f(t);x)$,

$$\tau_{n,r}(f(t);x)-f(t) = \frac{1}{P_n}\sum_{k=0}^{n} p_k\left\{s_{k,r}(f(t);x)-f(t)\right\}$$

$$= \frac{1}{2\pi P_n}\int_0^{\pi} \psi(t)\sum_{k=0}^{n}\frac{p_k}{k+1}\frac{\cos(rt)-\cos(k+r+1)t}{2\sin^2\frac{t}{2}}dt$$

$$= \frac{1}{2\pi P_n}\left[\int_0^{\pi/p_n} + \int_{\pi/p_n}^{\pi}\right]\psi(t)\left[\sum_{k=0}^{n}\frac{p_k}{k+1}\frac{\sin(k+2r+1)\frac{t}{2}\sin(k+1)\frac{t}{2}}{2\sin^2\frac{t}{2}}\right]dt$$

$$= I_1 + I_2 \quad \text{(say)}. \tag{4.1}$$

$$I_1 = \frac{1}{2\pi P_n}\int_0^{\pi/P_n}\psi(t)\left[\sum_{k=0}^{n}\frac{p_k}{k+1}\frac{\sin(k+2r+1)\frac{t}{2}\sin(k+1)\frac{t}{2}}{2\sin^2\frac{t}{2}}\right]dt$$

Using Hölder's inequality, $f(x)\in \mathrm{Lip}(\alpha,s) \Rightarrow \psi(t)\in \mathrm{Lip}(\alpha,s)$ on $[0,\pi]$ (McFadden [8], Lemma 5.40), equation (5), the fact that

$$(\sin t)^{-1} \leqslant \frac{\pi}{2t}, \quad 0<t\leqslant \pi/2,$$

$$|\sin nt| \leqslant n|\sin t|, \quad \forall t\in R, n\in N,$$

$r^{-1}+s^{-1}=1$, such that $1\leqslant r\leqslant\infty$, we get,

$$|I_1| \leqslant \frac{1}{2\pi P_n}\left\{\int_0^{\pi/P_n}\left(\frac{t\,|\psi(t)|}{t^{\alpha}}\right)^r dt\right\}^{1/r}\times$$

$$\times\left[\left\{\int_0^{\pi/P_n}\left(\frac{1}{t^{1-\alpha}}\left|\sum_{k=0}^{n}\frac{p_k}{k+1}\frac{\sin(k+2r+1)\frac{t}{2}\sin(k+1)\frac{t}{2}}{2\sin^2\frac{t}{2}}\right|\right)^s\right\}dt\right]^{1/s}$$

$$= O\left(\frac{1}{P_n}\right) O\left(\frac{1}{P_n}\right) \left\{ \left(\int_0^{\pi/p_n} \left(\frac{1}{t^{1-\alpha}} \sum_{k=0}^{n} p_k \frac{1}{t} \right)^s dt \right)^{1/s} \right\}$$

$$= O\left(\frac{1}{P_n}\right) O\left\{ \left(\int\limits_0^{\pi/p_n} t^{\alpha s - 2s} \right)^{1/s} \right\}$$

$$= O\left(\frac{1}{P_n}\right) O\left\{ \frac{1}{p_n^{\alpha-2+\frac{1}{s}}} \right\} = O\left\{ \frac{1}{p_n^{\alpha-2+\frac{1}{s}}} \right\}$$

$$= O\left\{ P_n^{-\alpha+\frac{1}{r}} \right\}.$$

Now by Hölder's inequality, we obtain

$$| I_2 | \leqslant O\left(\frac{1}{P_n}\right) \left\{ \int_{\pi/P_n}^{\pi} \left(\frac{t^{-\delta} |\psi(t)|}{t^{\alpha}} \right)^r dt \right\}^{1/r}$$

$$\left[\left\{ \int\limits_{\pi/P_n}^{\pi} \left(t^{\delta+\alpha} \left| \sum_{k=0}^{n} \frac{p_k}{k+1} \frac{\sin(k+2r+1)\frac{t}{2}\sin(k+1)\frac{t}{2}}{2\sin^2\frac{t}{2}} \right| \right)^s \right\} dt \right]^{1/s}$$

Now again using $f(x) \in \mathrm{Lip}(\alpha, s)$ it follows $\psi(t) \in \mathrm{Lip}(\alpha, s)$ on $[0, \pi]$ (McFadden [8], lemma 5.40), equation (6), the fact that

$$(\sin t)^{-1} \leqslant \frac{\pi}{2t}, \quad 0 < t \leqslant \pi/2,$$

$$|\sin nt| \leqslant n |\sin t|, \quad \forall t \in R, n \in N,$$

$r^{-1} + s^{-1} = 1$, such that $1 \leqslant r \leqslant \infty$, we find

$$I_2 = O\left(\frac{1}{P_n}\right) O\left\{ \int_{\pi/P_n}^{\pi} \left(\frac{t^{-\delta} |\psi(t)|}{t^{\alpha}} \right)^r dt \right\}^{1/r} \times$$

$$\times \left[\left\{ \int\limits_{\pi/P_n}^{\pi} \left(\frac{1}{t^{-\delta-\alpha}} \sum_{k=0}^{n} \frac{| p_k \sin(k+2r+1)\frac{t}{2} | \, (k+1) \, | \sin\frac{t}{2} |}{(k+1)\sin^2\frac{t}{2}} \right)^s \right\} dt \right]^{1/s}$$

$$= O\left(\frac{1}{P_n}\right) O\left\{ \left(\frac{1}{P_n}\right)^{-\delta} \right\} O\left\{ \left(\int_{\pi/P_n}^{\pi} \left(\frac{t^{\alpha+\delta}}{\sin\frac{t}{2}} \sum_{k=0}^{n} \left| p_k \sin(k+2r+1)\frac{t}{2} \right| \right)^s dt \right)^{1/s} \right\}$$

$$= O\left\{\left(\frac{1}{P_n}\right)^{1-\delta}\right\} O\left\{\left(\int_{\pi/P_n}^{\pi} t^{\alpha s+\delta s-2s} dt\right)^{1/s}\right\}.$$

Since $\{p_n\}$ is monotonic increasing, we find

$$\left|\sum_{k=0}^{n} p_k \sin(k+2r+1)\frac{t}{2}\right| \leqslant p_n \max_{0\leqslant\mu\leqslant n} \sum_{k=0}^{\mu} \sin(k+2r+1)\frac{t}{2} = O\left(\frac{p_n}{t}\right),$$

by virtue of the fact $\sum_{k=\lambda}^{\mu} e^{-ikt} = O(t^{-1})$, $0 \leqslant \lambda \leqslant \mu \leqslant \nu$.

Thus, we have

$$| I_2 | = O\left\{\left(\frac{1}{P_n}\right)\right\}^{1-\delta} O\left\{\left(\frac{1}{P_n}\right)\right\}^{\alpha+\delta-2+\frac{1}{s}} = O\left\{P_n^{-\alpha+\frac{1}{r}}\right\}$$

Now, using the L_r-norm, we get

$$\|\tau_{n,r}(f(t);x) - f(t)\|_r = \left\{\int_0^{2\pi} |\tau_{n,r}(f(t);x) - f(t)|^r dx\right\}^{1/r}$$

$$= O\left\{\int_0^{2\pi} P_n^{-r\alpha+r/r} dx\right\}^{1/r} = O\left\{P_n^{-\alpha+1/r}\right\}.$$

This completes the proof of Theorem 1.

5. Applications

The theory of approximation is a very extensive field, which has various applications. As mentioned in [19], the L_p - space in general, L_2 and L_∞ in particular play an important role in the theory of signals and filters. From the point of view of the applications, sharper estimates of infinite matrices are useful to get bounds for the lattice norms (which occur in solid state physics) of matrix valued functions, and enables to investigate perturbations of matrix valued functions and compare them.

The following corollaries can be derived from our Theorem 2.1.

COROLLARY 1. *If $r \to \infty$ in Theorem 1, then for $0 < \alpha < 1$, $f(x) \in \operatorname{Lip}\alpha$ and hence the degree of approximation by almost Riesz means $\tau_{n,r}$ of its infinite Fourier series is given by*

$$|\tau_{n,r}(f(t);x) - f(t)| = O\left(P_n^{-\alpha}\right).$$

Proof. For $r \to \infty$, we obtain

$$\|\tau_{n,r}(f(t);x) - f(t)\|_\infty = \sup_{0\leqslant x\leqslant 2\pi} |\tau_{n,r}(f(t);x) - f(t)| = O\left(P_n^{-\alpha}\right).$$

Thus we get

$$\begin{aligned} |\tau_{n,r}(f(t);x) - f(t)| &\leqslant \|\tau_{n,r}(f(t);x) - f(t)\|_\infty \\ &= \sup_{0\leqslant x\leqslant 2\pi} |\tau_{n,r}(f(t);x) - f(t)| \\ &= O(P_n^{-\alpha}). \end{aligned}$$

This completes the proof of Corollary 1. □

6. Conclusion

Several interesting results pertaining to the degree of approximation of periodic signals (functions) belonging to the Lipschitz classes by almost convergent sequences have been reviewed. Further, a proper set of conditions have been discussed to rectify the errors. The theorem of this paper is an attempt to formulate the problem of approximation of signal (function) Lip(α,r), $(r \geqslant 1)$ through trigonometric polynomials generated by the almost Riesz transform of the Fourier series of f in a simpler manner. Some interesting applications of the approximation of signals used in this manuscript have also been highlighted. Also, the result of our theorem is more general rather than the results of any other previous proved theorems, which will be enrich the literate of summability theory of infinite series.

Competing interests The authors declare that there is no conflict of interests regarding the publication of this paper.

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