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# LETTER TO THE EDITOR

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*Abstract.* In the note some remarks are given concerning certain results of the article [M. A. Sarigöl, *A remark on* $\varphi - |\overline{N}, q_n; \delta|_k$ *summability*, this journal, **12**, 1 (2018), 55–58.]

## 1. Introduction

Let $\sum a_n$ be a given infinite series with partial sums $(s_n)$. Let $(p_n)$ be a sequence of positive numbers for which there holds

$$P_n = \sum_{v=0}^{n} p_v \longrightarrow \infty (n \to \infty), \qquad (P_{-i} = p_{-i} = 0, \quad i \geqslant 1). \tag{1.1}$$

The sequence-to-sequence transformation

$$\sigma_n = \frac{1}{P_n} \sum_{v=0}^{n} p_v s_v \tag{1.2}$$

defines the sequence $(\sigma_n)$ of the Riesz mean or simply the $(\overline{N}, p_n)$ mean of the sequence $(s_n)$ generated by the sequence of coefficients $(p_n)$ [3]. The series $\sum a_n$ is said to be summable $|\overline{N}, p_n; \delta|_k$, $k \geqslant 1$ and $\delta \geqslant 0$ if [2]

$$\sum_{n \geqslant 1} \left( \frac{P_n}{p_n} \right)^{(\delta+1)k-1} |\sigma_n - \sigma_{n-1}|^k \tag{1.3}$$

converges. In turn, setting $\delta = 0$, we obtain the $|\overline{N}, p_n|_k$, $k \geqslant 1$ summability, see [1].

Let $(\varphi_n)$ be any sequence of positive reals. The series $\sum a_n$ is summable in the sense $\varphi - |\overline{N}, p_n; \delta|_k$, $k \geqslant 1$ and $\delta \geqslant 0$ when [7]

$$\sum_{n \geqslant 1} \varphi_n^{(\delta+1)k-1} |\sigma_n - \sigma_{n-1}|^k < \infty. \tag{1.4}$$

If we specialize $\delta = 0$ and $\varphi_n = P_n / p_n$, then $\varphi - |\overline{N}, p_n; \delta|_k$ summability reduces to $|\overline{N}, p_n|_k$ summability.

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## 2. Known results

In [4] Özarslan proved the following theorems to obtain equivalence between two general summability methods.

THEOREM 2.1. *Let* $k \geqslant 1$ *and* $0 \leqslant \delta < 1/k$ *while* $(\varphi_n), (p_n), (q_n)$ *be positive sequences. Assume that*

$$\sum_{n=\nu+1}^{m+1} \frac{\varphi_n^{(\delta+1)k-1} q_n^k}{Q_n^k Q_{n-1}} = \mathscr{O}\left(\varphi_\nu^{(\delta+1)k-1} \frac{q_\nu^{k-1}}{Q_\nu^k}\right) \tag{2.1}$$

*as* $m \to \infty$. *In order that every* $\varphi - |\overline{N}, p_n; \delta|_k$ *summable series be* $\varphi - |\overline{N}, q_n; \delta|_k$ *summable it is necessary that there holds*

$$\frac{q_n P_n}{Q_n p_n} = \mathscr{O}(1). \tag{2.2}$$

*Moreover, when*

$$\frac{p_n Q_n}{P_n q_n} = \mathscr{O}(1), \tag{2.3}$$

*then* (2.2) *is also sufficient for the asserted conclusion.*

THEOREM 2.2. *Let* $(p_n)$ *and* $(q_n)$ *be positive sequences satisfying the constraint* (2.1), $k \geqslant 1$ *and* $0 \leqslant \delta < 1/k$. *In order that* $\varphi - |\overline{N}, p_n; \delta|_k$ *is equivalent to* $\varphi - |\overline{N}, q_n; \delta|_k$ *summability it is necessary and sufficient that both* (2.2) *and* (2.3) *hold.*

## 3. Remarks upon results

Firstly, reference [5] (listed also in [6]) concerns Fourier series which are not mentioned in [6]. Also, it should be noticed that in [4], Özarslan has not tried to establish both necessary and sufficient conditions in order that every $\varphi - |\overline{N}, p_n; \delta|_k$ summable series should be $\varphi - |\overline{N}, q_n; \delta|_k$ summable; the article [4] contains the equivalence of two general summability methods *only under some suitable conditions*.

So, there is no relevance between [6] and [4].

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