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GENERALIZATIONS OF PICARD'S THEOREM WITH MOVING HYPERSURFACES

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Abstract. We generalize the classical Big Picard Theorem to holomorphic mappings of several complex variables into the complement of moving hypersurfaces in general position (NOT just point-wise general position) in $\mathbf{P}^n(\mathbf{C})$.

1. Introduction and the results

Let Δ_r denote the disc of radius r in the complex plane. For simplicity, we will denote the unit disc by Δ. Picard's big theorem and little theorem are related about the rang of meromorphic function, which are stated as follows:

THEOREM A. (Little Picard Theorem) *Every holomorphic map* $f : \mathbf{C} \to \mathbf{P}^1(\mathbf{C}) \setminus \{0,1,\infty\}$ *of the complex line into the Riemann sphere with three punctures is constant.*

THEOREM B. (Big Picard Theorem) *Every holomorphic map* $f : \Delta \setminus \{0\} \to \mathbf{P}^1(\mathbf{C}) \setminus \{0,1,\infty\}$ *of a punctured disk into a sphere with three punctures can be extended to a holomorphic map* $f : \Delta \to \mathbf{P}^1(\mathbf{C}) \setminus \{0,1,\infty\}$ *of the disk into the Riemann sphere.*

For the case of higher dimension, several generalizations of the big Picard theorem are obtained.Base on Kobayashi's fundamental work [11], Kiernan [10] generalized Big Picard Theorem to the following result.

THEOREM C. ([10]) *Let S be an analytic subset of the complex manifold M whose singularities are normal crossings and let X be a hyperbolically imbedded subspace of the complex space Y. Then any holomorphic map $f : M \setminus S \to X$ can be extended to a holomorphic map $f : M \to Y$.*

Fujimoto [4, 5] and Green [7] established the following Picard-type theorems.

THEOREM D. ([4, 7]) *Let f be a holomorphic mapping from $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. If f omits $2n+1$ hyperplanes in $\mathbf{P}^n(\mathbf{C})$ located in general positon, then f is constant.*

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THEOREM E. ([5]) *Let S be a regular thin analytic subset of a domain D in $\mathbf{C}^m$. Then every holomorphic mapping f of $D\setminus S$ into the complement of $2n+1$ hyperplanes in general position in $\mathbf{P}^n(\mathbf{C})$ can be extended to a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$.*

Later, Eremenko extended the above result to the case of hypersurfaces.

THEOREM F. ([2]) *Let X be a closed subset of $\mathbf{P}^n(\mathbf{C})$ and let $Q_1,\cdots,Q_{2t+1}$ be hypersurfaces in $\mathbf{P}^n(\mathbf{C})$ located in t-subgeneral position with respect to X. Then every holomorphic mapping $f:\mathbf{C}\to X\setminus\cup_{j=1}^{2t+1}Q_j$ is constant.*

In 2002, Noguchi and Winkelmann [13] obtained the result says that *Let $X\subseteq \mathbf{P}^n(\mathbf{C})$ be an irreducible subvariety of dimension k, and $Q_1,\ldots,Q_{2k+1}$ be distinct hypersurfaces cuts of X that are in general position as hypersurfaces of X. Then $X\setminus\cup_{j=1}^{2k+1}Q_j$ is complete hyperbolic and hyperbolically imbedded into X.* This together with Theorem C implies the following.

THEOREM G. *Let S be an analytic subset of a domain $D\subset\mathbf{C}^m$, whose singularities are normal crossings. Let $X\subseteq\mathbf{P}^n(\mathbf{C})$ be an irreducible subvariety of dimension k. Let $Q_1,\cdots,Q_{2k+1}$ be $2k+1$ hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ on D located in general position with respect to X. Let f be a holomorphic mapping of $D\setminus S$ into X. If f omits each Q_j for all $1\leqslant j\leqslant 2k+1$, then f can be extended to a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$.*

Furthermore, there is a natural problem: *Are the Picard type theorems valid for holomorphic mappings involving moving targets ?* Our main result will be given after we fix some notation and definition.

Let Q be a fixed hypersurface of degree d in $\mathbf{P}^n(\mathbf{C})$, which is defined by a homogeneous polynomial $P(x_0,\ldots,x_n)\in\mathbf{C}[x_0,\ldots,x_n]$, i. e.

$$Q=\{[w_0:\cdots:w_n]\in\mathbf{P}^n(\mathbf{C});\ P(w_0,\ldots,w_n)=0\}.$$

Denote by $\mathscr{H}_D$ the ring of all holomorphic functions on D. For any positive integer number d, set

$$\mathscr{T}_d=\{(i_0,\ldots,i_n)\in\mathbb{N}^{n+1};\ i_0+\cdots+i_n=d\}.$$

A moving hypersuface (on D) $Q(z)$ be of degree d in $\mathbf{P}^n(\mathbf{C})$ generalize, to every $z_0\in D$, a fixed hypersurface given by

$$Q(z_0)=\Big\{[w_0:\cdots:w_n]\in\mathbf{P}^n(\mathbf{C});\ \sum_{(i_0,\ldots,i_n)\in\mathscr{T}_d}a_{i_0\cdots i_n}(z_0)w_0^{i_0}\cdots w_n^{i_n}=0\Big\},$$

where the coefficients $a_{i_0\cdots i_n}(z)$ are holomorphic functions on D without common zeros.

DEFINITION 1. Let $Q_1(z),\cdots,Q_q(z)$ $(q\geqslant t+1)$ be moving hypersurfaces in $\mathbf{P}^n(\mathbf{C})$ and $X\subseteq\mathbf{P}^n(\mathbf{C})$ be a closed set. We say that moving hypersurfaces are *in pointwise t-subgeneral position with respect to X,* if for each $z\in D$, the fixed hypersurfaces $Q_1(z),\cdots,Q_q(z)$ are in t-subgeneral position with respect to X. We say that moving persurfaces are *in t-subgeneral position with respect to X*, if there exists $z_0 \in D$ such that the fixed hypersurfaces $Q_1(z_0), \cdots, Q_q(z_0)$ are in t-subgeneral position with respect to X.

Unfortunately, when the hyperplanes are moving, Little Picard theorems are not valid even in case the moving hyperplanes are in pointwise position. This can be seen easily from the nonconstant holomorphic mapping $f(z) = [1 : \exp(z) : \exp(2z)]$ of $\mathbf{C}$ into $\mathbf{P}^2(\mathbf{C})$ which omits the moving hyperplanes $H_1(z) = \{x_0 = 0\}$, $H_2(z) = \{x_1 = 0\}$, $H_3(z) = \{x_2 = 0\}$, $H_4(z) = \{\exp(z)x_0 + x_1 + \exp(-z)x_2 = 0\}$, $H_5(z) = \{x_0 + 2\exp(-z)x_1 + 3\exp(-2z)x_2 = 0\}$ located in pointwise general position on $\mathbf{C}$.

On the other hand, the Big Picard Theorem was extended to the case of holomorphic mappings into the complement of moving hypersurfaces in pointwise general position.

THEOREM H. ([8]) *Let S be an analytic subset of a domain $D \subset \mathbf{C}^m$ with codimension one, whose singularities are normal crossings. Let f be a holomorphic mapping of $D \setminus S$ into $\mathbf{P}^n(\mathbf{C})$. Assume that f omits $2n+1$ moving hypersurfaces $\{Q_j(z)\}_{j=1}^{2n+1}$ in $\mathbf{P}^n(\mathbf{C})$ in point-wise general position on D. Then f can be extended to a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$.*

THEOREM I. ([14]) *Let f be a holomorphic mapping of a domain $D \setminus S$ into X, where D is a domain in $\mathbf{C}^m$, S is an analytic subset of D of codimension one whose singularities are normal crossings, and X is an irreducible subvariety of $\mathbf{P}^n(\mathbf{C})$. Let $Q_1(Z), \cdots, Q_q(Z)$ be q moving hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ on D located in pointwise subgeneral position with respect to X. Assume that f does not intersect each $Q_j(Z)$ on $D \setminus S$ for all $1 \leqslant j \leqslant q$. If $q \geqslant 2\dim X + 1$, then f can be extended to a holomorphic mapping from D into $\mathbf{P}^n(\mathbf{C})$.*

Following this line, in in this paper we prove some big Picard-type theorems for holomorphic mappings into the complement of moving hypersurfaces in general position in the complex projective space.

THEOREM 1. *Let S be an analytic subset of a domain $D \subset \mathbf{C}^m$ with codimension one, whose singularities are normal crossings. Let $X \subseteq \mathbf{P}^n(\mathbf{C})$ be an irreducible subvariety of dimension k. Let $Q_1(Z), \cdots, Q_{2k+1}(Z)$ be $2k+1$ moving hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ on D located in general position with respect to X. Let f be a holomorphic mapping of D into X. If f omits each $Q_j(Z)$ on $D \setminus S$ for all $1 \leqslant j \leqslant 2k+1$, then f can be extended to a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$.*

2. Preliminaries

In order to prove Theorem 1, we need some preparations.

DEFINITION 2. Let D be a domain in $\mathbf{C}^m$. A family $\mathscr{F}$ of holomorphic mappings of D into $\mathbf{P}^n(\mathbf{C})$ is said to be *normal* if $\mathscr{F}$ is relatively compact in $Hol(D, \mathbf{P}^n(\mathbf{C}))$ in the compact-open topology.

Let D be a domain in $\mathbf{C}^m$. If $z \in D$ and $\xi \in \mathbf{C}^m$, then we define the infinitesimal form of the *Kobayashi pseudo-metric* for D at z in the direction ξ to be

$$F_K^{\Omega}(z,\xi) = \inf\left\{\frac{\|\xi\|}{\|f'(0)\|};\ f:\Delta \to D,\ f(0)=z,\ f'(0) \text{ is a constant multiple of } \xi\right\}.$$

Here $\|\cdot\|$ represents Euclidean length.

DEFINITION 3. Let Ω be a hyperbolic domain in $\mathbf{C}^m$ and let M be a complete complex Hermitian manifold with metric ds_M^2. A holomorphic map f from Ω into M is said to be *normal* if there exists a positive constant c such that for all $z \in \Omega$ and all $\xi \in T_z(\Omega)$,

$$|ds_M^2(f(z), df(z)(\xi))| \leqslant cF_K^{\Omega}(z,\xi),$$

where $df(z)$ is the tangent mapping from $T_z(\Omega)$ into $T_{f(z)}(M)$.

LEMMA 1. ([1]) *If f is a holomorphic map from a hyperbolic domain Ω in $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$, then f is normal if and only if the sequence $\{f\circ\varphi;\ \varphi \in Hol(\Delta;\Omega)\}$ is a normal family.*

We need the following lemma, which is an extension of Zalcman Lemma.

LEMMA 2. ([1]) *Let $\mathscr{F}$ be a family of holomorphic maps of a domain D in $\mathbf{C}^m$ into $\mathbf{P}^n(\mathbf{C})$. The family $\mathscr{F}$ is not normal on D if and only if there exist sequences $\{f_\nu\}_{\nu=1}^{\infty} \subset \mathscr{F}$, $\{z_\nu\}_{\nu=1}^{\infty} \subset D$ with $z_\nu \to z_0 \in D$, $\{\rho_\nu\}_{\nu=1}^{\infty}$ with $\rho_\nu > 0$ and $\rho_\nu \to 0$ and $\{u_\nu\} \subset \mathbb{C}^m$ Euclidean unit vectors, such that*

$$F_\nu(\xi) := f_\nu(z_\nu + \rho_\nu u_\nu \xi)$$

converges uniformly on compact subsets of $\mathbf{C}$ to a nonconstant holomorphic mapping $F(\xi)$ of $\mathbf{C}$ into $\mathbf{P}^n(\mathbf{C})$.

Next, we provide the following lemma by using Theorem 2.3 in [9].

LEMMA 3. ([9]) *Let S be an analytic subset of a domain Ω in $\mathbf{C}^m$ with codimension one, whose singularities are normal crossings. Let f be a holomorphic map of $\Omega\setminus S$ into $\mathbf{P}^n(\mathbf{C})$. If f is normal then it can be extended to a holomorphic map of Ω into $\mathbf{P}^n(\mathbf{C})$.*

Throughout this paper, let f be a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$ and let $Q(z)$ be a moving hypersurface of $\mathbf{P}^n(\mathbf{C})$ on D with degree d defined by

$$\sum_{(i_0,\dots,i_n)\in\mathscr{T}_d} a_{i_0\cdots i_n}(z) w_0^{i_0}\cdots w_n^{i_n} = 0.$$

For any reduced representation $\mathbf{f} = (f_0,\dots,f_n)$ of f on $U(\subset D)$, we define the holomorphic function on U

$$\langle \mathbf{f}, Q(z)\rangle := a_{i_0\cdots i_n}(z) f_0^{i_0}\cdots f_n^{i_n},$$

Moreover, we write f instead of $\mathbf{f}$ when the properties are independent of the choice of a reduced representation, for example, we can consider the function $\frac{f_0(z)^d}{\langle f(z), Q\rangle}$.

3. Proofs

First, we prove the following lemma which plays a key role in the proof of our theorems.

LEMMA 4. *Let f be a holomorphic map from a bounded domain D in $\mathbf{C}^m$ into X, where $X \subseteq \mathbf{P}^n(\mathbf{C})$ is an irreducible subvariety of dimension k. Let $\{Q_j(Z)\}_{j=1}^{2k+1}$ be moving hypersurfaces of $\mathbf{P}^n(\mathbf{C})$ on $\overline{D}$ located in general position with respect to X. If f omits each $Q_j(Z)$ on D for all $1 \leqslant j \leqslant 2k+1$. Then f is a normal map.*

Proof. Set

$$S := \{Z \in \overline{D};\ \{Q_j(Z)\}_{j=1}^{2k+1} \text{ are not in general position}\}.$$

Then S is either a thin analytic subset of $\overline{D}$ or an empty set. Suppose that f is not a normal map, then, by Lemma 1, the family $\mathscr{F} := \{f \circ \varphi;\ \varphi \in Hol(\Delta; D)\}$ is not normal. According to Lemma 2, there exist a sequence $\{f_v\}_{v=1}^{\infty}$ in $\mathscr{F}$, points $z_v \to z_0 \in \Delta$, and positive number $\rho_v \to 0$, such that

$$F_v(\xi) := f_v(z_v + \rho_v \xi) = f \circ \varphi_v(z_v + \rho_v \xi)$$

where $\xi \in \mathbf{C}$ satisfies $z_v + \rho_v \xi \in \Delta$, converges uniformly on compact subsets of $\mathbf{C}$ to a nonconstant holomorphic mapping $F(\xi)$ of $\mathbf{C}$ into $\mathbf{P}^n(\mathbf{C})$. We write $f_v = f \circ \varphi_v$ where $\varphi_v = (\varphi_{v1}, \ldots, \varphi_{vm}) \in Hol(\Delta; D)$. Since D is a bounded domain and $\varphi_v \in Hol(\Delta; D)$, we have each sequence $\{\varphi_{vi}\}_{v=1}^{\infty}$ is a normal family for $i = 1, 2, \ldots, m$. Without loss of generality, we assume that $\{\varphi_v(z)\}$ converges uniformly on compact subsets of Δ to a holomorphic mapping $\varphi_0(z)$ of Δ into $\overline{D}$.

Set $Z_0 = \varphi_0(z_0) \in \overline{D}$, then the sequence $\{\varphi_v(z_0)\}$ converges to Z_0. Since f does not intersect $Q_j(Z)$, we conclude that either F does not intersect $Q_j(Z_0)$ or $F(\mathbf{C})$ is included in $Q_j(Z_0)$ for all $1 \leqslant j \leqslant 2n+1$ according to the Hurwitz theorem. Hence, there is a subset I of $\{1, \ldots, 2n+1\}$ such that $i \in I$ iff $F(\mathbf{C}) \subseteq Q_i(Z_0)$. Then we have

$$F(\mathbf{C}) \subseteq \big(\cap_{i \in I} Q_i(Z_0) - \cap_{i \notin I} Q_i(Z_0)\big).$$

We separate two cases:

Case 1. The fixed hypersurfaces $\{Q_j(Z_0)\}_{j=1}^{2k+1}$ in $\mathbf{P}^n(\mathbf{C})$ are in general position.

If $I = \emptyset$, we have $F \in Hol(\mathbf{C}, X \setminus \cup_{j=1}^{2k+1} Q_j(Z_0))$. It follows Theorem B that $F(\xi)$ is constant. This leads to a contradiction. So we may suppose that $I \neq \emptyset$ and consider the close set $V := \cap_{j \in I} Q_j(Z_0)$ in $\mathbf{P}^n(\mathbf{C})$. By the assumption that $\{Q_j(Z_0)\}_{j=1}^{2n+1}$ are in general position, we see that the hypersurfaces $\{Q_j(Z_0)\}_{j \notin I}$ locate in $k-l$ subgeneral position with respect to V, where $l = \#I$. So $F \in Hol(\mathbf{C}, V \setminus \{Q_j(Z_0)\}_{j \notin I})$. Note that $2k+1 > 2(k-l)+1$, we have the conclusion is now evident from Theorem F again.

Moreover, by the proof of Case 1 we get $\{f_v\}$ is a normal family on $\Delta \setminus \varphi_0^{-1}(S)$.

Case 2. The fixed hypersurfaces $\{Q_j(Z_0)\}_{j=1}^{2k+1}$ in $\mathbf{P}^n(\mathbf{C})$ are not in general position.

By the usual diagonal argument, we can find a subsequence of $\{f_\nu\}_{\nu=1}^{\infty}$ (again denoted by $\{f_\nu\}_{\nu=1}^{\infty}$) which converges uniformly on compact subset of $\Delta\setminus\varphi_0^{-1}(S)$ to a holomorphic mapping h. Without loss of generality, we may assume that $h(\Delta\setminus\varphi_0^{-1}(S))\nsubseteq Q_1(\varphi_0(z))$.

Assume that the degree of the bypersurface Q_1 is d_1 and each f_ν have a reduced representation

$$\mathbf{f}_\nu(z)=(f_{\nu 0}(z),\dots,f_{\nu n}(z)),\quad z\in\Delta$$

for all $\nu=1,2,\dots$. We define a holomorphic mapping g_ν of Δ into $\mathbf{P}^{n+1}(\mathbf{C})$ induced by the mapping

$$g_\nu(z)=\left[1:\ \frac{f_{\nu 0}(z)^{d_1}}{\langle\mathbf{f}_\nu(z),Q_1(\varphi_\nu(z))\rangle}:\cdots:\frac{f_{\nu n}(z)^{d_1}}{\langle\mathbf{f}_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right]:\ \Delta\to\mathbf{P}^{n+1}(\mathbf{C})$$

for $\nu=1,2,\dots$. The above definition is independent of the choice of the reduced representation of f_ν. Correspondingly, we obtain

$$G_\nu(\xi):=g_\nu(z_\nu+\rho_\nu\xi)$$

converges uniformly on compact subsets of $\mathbf{C}$ to a holomorphic mapping G of $\mathbf{C}$ into $\mathbf{P}^{n+1}(\mathbf{C})$. If we take a reduced representation

$$\mathbf{F}(\xi)=(F_0(\xi),\dots,F_n(\xi))$$

of F on $\mathbf{C}$, then we can obtain a reduced representation of G on $\mathbf{C}$ as follow:

$$\mathbf{G}(\xi)=(\langle\mathbf{F},Q_1(Z_0)\rangle(\xi),\ F_0^{d_1}(\xi),\dots,F_n^{d_1}(\xi)).$$

Since F is nonconstant, so is G. Thus, it follows from Lemma 2 that $\{g_\nu\}$ is not a normal family on Δ.

Set $A:=\varphi_0^{-1}(S)\cup h^{-1}(Q_1(\varphi_0(z))$ and $E:=\varphi_0(A)$. Since $h^{-1}(Q_1(\varphi_0(z))$ is a discrete point set on Δ, we deduce that $E=S\cup\varphi_0(h^{-1}(Q_1(\varphi_0(z)))$ is a thin analytic set on $\mathbf{C}^m$.

Fix any $z\in\Delta$, one can always find neighborhoods U and V of $Z=\varphi_0(z)$ such that $V\subset U\Subset\mathbf{C}^m$ and $(\overline{U}-V)\cap E=\emptyset$, i. e., $\left(\varphi_0^{-1}(U)-\varphi_0^{-1}(V)\right)\cap A=\emptyset$. Since $Z\in V\subset U$, we have that $z\in\varphi_0^{-1}(V)\subset\varphi_0^{-1}(U)$ (see [6, pp. 28 the proof for Proposition 3.5]).

Now we take the connected component contains z W_1 in $\varphi_0^{-1}(U)$ and set $W_2:=W_1\cap\varphi_0^{-1}(V)$. Therefore, $z\in W_2\subset W_1$, $(W_1-W_2)\cap A=\emptyset$. For our purpose, it suffices to show that $\left\{\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right\}_{\nu=1}^{\infty}$ converges uniformly on the open set W_1 of z. By the assumptions, the sequence of holomorphic functions $\left\{\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right\}_{\nu=1}^{\infty}$ converges uniformly to $\frac{h_i^{d_1}(z)}{\langle h(z),Q_1(\varphi_0(z))\rangle}$ on W_1-W_2. Thus, if

$$M:=\sup_{z\in W_1-W_2}\left|\frac{h_i^{d_1}(z)}{\langle h(z),Q_1(\varphi_0(z))\rangle}\right|,$$

then

$$\sup_{z\in W_1-W_2}\left|\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right|\leqslant M+1$$

for sufficiently large ν.

Notice that the functions $\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}$ are holomorphic on W_1, so we have

$$\sup_{z\in W_1}\left|\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right|=\sup_{z\in W_1-W_2}\left|\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right|$$

by Maximum Modulus Principle. Therefore, for $0\leqslant i\leqslant n$, we get that $\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}$ are uniformly bounded on W_1, whence by Montel's criterion, we have that there is a subsequence of $\left\{\frac{f_{\nu i}^{d_1}(z)}{\langle f_\nu(z),Q_1(\varphi_\nu(z))\rangle}\right\}_{\nu=1}^{\infty}$ which is converges uniformly to a holomorphic function on W_1. And thus, $\{g_\nu\}$ is a normal family on Δ. Contradiction. The proof is completed. $\square$

We now proof Theorem 1.

Proof. For $Z_0\in S$, we take a bounded domain $\Omega_0(\Subset D)$ containing Z_0. It is easy to see that $S\cap\Omega_0$ is an analytic subset of the domain Ω_0 of codimension one whose singularities are normal crossings. It follows from Lemma 4 that f is a normal map on $\Omega_0\setminus S$. Therefore, f can be extended to a holomorphic map of Ω_0 into $\mathbf{P}^n(\mathbf{C})$ by virtue of Lemma 3. The proof is completed. $\square$

By using Lemma 4 and suitable modification to the proof of Theorem 1, we can obtain the following Theorem. The details of the proof will be omitted.

THEOREM 2. *Let D be a domain in $\mathbf{C}^m$ and let $S\subset D$ be either a closed analytic subset with codimension at least two or a closed subset with $(2m-2)$-dimensional Hausdorff measure equal to zero. Let f be a holomorphic mapping of $D\setminus S$ into $\mathbf{P}^n(\mathbf{C})$. Assume that f omits $2n+1$ moving hypersurfaces $\{Q_j(z)\}_{j=1}^{2n+1}$ in $\mathbf{P}^n(\mathbf{C})$ in general position on D. Then f can be extended to a holomorphic mapping of D into $\mathbf{P}^n(\mathbf{C})$.*

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