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UNICITY OF SHIFT POLYNOMIALS GENERATED BY MEROMORPHIC FUNCTIONS

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Abstract. This paper aims to prove the uniqueness result for shift polynomials of a meromorphic function and its higher order derivative sharing polynomials under suitable conditions. The result obtained generalizes the existing literature and examples given prove the acuteness of the imposed conditions.

1. Introduction

Throughout this article, the phrase "meromorphic function" means that the function is analytic everywhere except for poles in $\mathbb{C}$ and "entire function" means that the function is analytic everywhere in $\mathbb{C}$. The fundamentals of Nevanlinna theory can be read in [3, 7, 18]. Denote the set $E = \{x : x \in \mathbb{R}^+\}$. Let $\mathscr{F} = \{f : f$ *is non-constant meromorphic function in* $\mathbb{C}\}$. For $f, g \in \mathscr{F}$ and $b \in \mathbb{C} \cup \{\infty\}$, if $f - b$ and $g - b$ have the identical zeros including multiplicities then f and g share b CM (counting multiplicities), if the multiplicities are ignored, then f and g share b IM (ignoring multiplicities) and if $1/f$ and $1/g$ share 0 CM then, f and g share ∞ CM [19]. We call b as the b points of f and g or value points of f and g. For $\phi(z) \in \mathscr{F}$, if $T(r,\phi) = S(r,f)$ then ϕ is called the "small function" of f where $T(r,\phi)$ is the Nevanlinna characteristic function and $S(r,f) = o(T(r,f))$, $r \notin E$, $r \to \infty$.

DEFINITION 1. [6] Let $E_k(b;f)$ denote the set of all b points of f. The multiplicity m of b is counted m times if $m \leqslant k$ and is counted $k+1$ times if $m > k$. If $E_k(b;f) = E_k(b;g)$ then f, g share b with weight k.

Throughout this article, $F_* = f^n$ and by the sentence f, g shares (b,k) means that f, g shares the value b with weight k. f, g shares $(b,0)[(b,\infty)] \Longleftrightarrow f, g$ shares b IM[(CM)].

DEFINITION 2. [17]

$$N\left(r,\frac{1}{f}\right) = \int_0^r \frac{n(t,f) - n(0,f)}{t}\,dt + n(0,f)\,\log r,$$

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wherein $n(r,f)$ denotes the number of zeros of f in the interior of the disk $|z|<r$ and $N\left(r,\frac{1}{f}\right)$ is called as the counting function of f.

Let $q\in\mathbb{Z}^+$ then $N_q(r,\frac{1}{f-b})$ denotes the counting function of f whose b-points are counted with the multiplicity q, the counting function $N_{(q}(r,\frac{1}{f-b})$ of f means those b-points counted with proper multiplicity whose multiplicities are greater than q and $N_{q)}(r,\frac{1}{f-b})$ denotes the counting function of f whose b-points counted with proper multiplicity where the multiplicities are less than q. Correspondingly the reduced counting functions are given by $\overline{N}_q(r,\frac{1}{f-b}),\overline{N}_{(q}(r,\frac{1}{f-b})$ and $\overline{N}_{q)}(r,\frac{1}{f-b})$ where the multiplicities are ignored [15]. The following definitions play a major role in the understanding of the main result:

DEFINITION 3. [16] Let $f\in\mathscr{F}$. The "order of f" is given by

$$\rho(f)=\limsup_{r\to\infty}\frac{\log^+T(r,f)}{\log r}. \tag{1}$$

DEFINITION 4. [5]

$$N_k\left(r,\frac{1}{f-b}\right)=\overline{N}\left(r,\frac{1}{f-b}\right)+\sum_{j=2}^{k}\overline{N}_{(j}\left(r,\frac{1}{f-b}\right).$$

In 1920, R. Nevanlinna stated that if two entire functions f and g share five distinct values IM then the functions are identical or unique and the condition of sharing five values is inevitable. From then on researchers round the globe have extended this uniqueness result in various prospective. Many authors have implemented the uniqueness result for class of meromorphic functions. In 2009, Zhang gave the following noteworthy result:

THEOREM 1. [20] *Let $f\in\mathscr{F}$ and $n(\geqslant 7)\in\mathbb{Z}$. If F_* and F_*' share 1 CM, then $F_*\equiv F_*'$, and f assumes the form $f(z)=ce^{(\frac{z}{n})}$, where $c\neq 0$ is a constant.*

Gradually the first derivative in the above result was extended to k^{th} derivative and the sharing of small function was introduced in the same article as follows:

THEOREM 2. [20] *Let $f\in\mathscr{F}$, $n,k\in\mathbb{Z}^+$ and $a(z)(\not\equiv 0,\infty)$ be a small function of f. If suppose F_*-a and $(F_*)^{(k)}-a$ share the value 0 CM and $(n-k-1)(n-k-4)>3k+6$, then $F_*\equiv(F_*)^{(k)}$ and f assumes the form $f(z)=c_1e^{(\frac{\lambda z}{n})}$ where c_1 is a non-zero constant and $\lambda^k=1$.*

The result of theorem 2 was obtained under different necessary condition for sharing of small function a CM and a IM in [21]. Following the trend many authors incorporated different sharing conditions for F_* and $F_*^{(k)}$ like polynomial sharing, two distinct polynomial sharing, set sharing [1] and so on. For related work, one can refer [4, 13, 14, 20]. Lahiri and Majumder in [13] proved the uniqueness theorem for F_* and $F_*^{(k)}$ sharing two distinct small functions by introducing the concept of weighted sharing. The result is noted below:

THEOREM 3. [4] *Let the transcendental function* $f \in \mathscr{F}$ *such that* $N(r,f) = S(r,f)$ *and* $a_i = a_i(z)(\not\equiv 0, \infty)$ *be small functions of* f, *where* $i = 1,2$. *Let* $n,k \in \mathbb{Z}^+$ *such that* $n \geqslant k+1$. *In addition if* $F_* - a_1$ *and* $(F_*)^{(k)} - a_2$ *share* $(0,1)$, *then* $(F_*)^{(k)} \equiv \frac{a_2}{a_1} F_*$. *Furthermore, if* $a_1 \equiv a_2$, *then the conclusion of theorem 2 holds.*

The results of theorem 1, theorem 2 and theorem 3 for the entire class of functions can be referred in [9, 10, 11, 20, 21, 22]. With the advent of difference analogue of Nevanlinna theory, uniqueness theorems have evolved in this regard as well. In this direction Majumder-Saha [12] gave the below stated result:

THEOREM 4. [12] *Let the transcendental function* $f \in \mathscr{F}$ *be of finite order with finitely many poles. For constant* $c(\neq 0) \in \mathbb{C}$, $n,k \in \mathbb{N}$, *let* $F_*(z) - Q_1(z)$ *and* $(F_*(z+c))^{(k)} - Q_2(z)$ *share* $(0,1)$ *and* $f(z)$, $f(z+c)$ *share* 0 *CM. If* $n \geqslant k+1$, *then* $(F_*(z+c))^{(k)} \equiv \frac{Q_2(z)}{Q_1(z)} F_*(z)$, *where* Q_1, Q_2 *are polynomials with* Q_1, $Q_2 \not\equiv 0$. *Furthermore, if* $Q_1 = Q_2$, *then the conclusion of theorem 2 holds.*

Now it is interesting to see what happens to the conclusion of theorem 4 when $f^n(z)P(f(z))$ and $[f^n(z+c)P(f(z+c))]^{(k)}$ replaces $F_*(z)$ and $(F_*(z+c))^{(k)}$. Taking a cue from this, the main result of the paper is stated below:

THEOREM 5. *Let* $f(z)$ *be a transcendental meromorphic function of finite order having poles of finite number. Define* $P(f) = \sum_{i=0}^{m} a_i f^i$ *such that* a_i, $i \in \{0,1,\cdots,m\}$ *are non zero constants. Let* c *be a non zero complex constant and* m, n, k *be positive integers. If* $[f^n(z)P(f(z))] - Q_1(z)$ *and* $[f^n(z+c)P(f(z+c))]^{(k)} - Q_2(z)$ *share* $(0,1)$ *such that* $f(z)$ *and* $f(z+c)$ *share* 0 *CM with* $n \geqslant m+k+1$ *then* $[f^n(z+c)P(f(z+c))]^{(k)} \equiv \frac{Q_2}{Q_1}[f^n(z)P(f(z))]$ *where* Q_1 *and* Q_2 *are non zero polynomials with* Q_1, $Q_2 \not\equiv 0$. *In addition if* $Q_1 \equiv Q_2$ *then* $f(z) = C\, e^{\frac{\lambda}{n+i}}$, $i \in \{0,1,\cdots,m\}$, C, λ *are constants such that* $e^{\lambda C} = 1$ *and* $\lambda^k = 1$.

REMARK 1. In the theorem 5 if $a_i = 0$, $i \in \{1,2,\cdots,m\}$ and $a_0 = 1$, then $P(f) = 1$, $m = 0$ and the conclusion of theorem 4 holds. Thus the main outcome of this paper is the generalization of result obtained in [12].

EXAMPLE 1. Let $f(z) = e^z + 1$, $c = 2\pi i$, $P(f) = f - 2$. Clearly $f(z)$ and $f(z+c)$ share 0 CM. For $Q_1 = 1$ and $Q_2 = 8$, $[f(z)P(f(z))] - Q_1(z)$ and $[f(z+c)P(f(z+c))]^{(2)} - Q_2(z)$ share 0 CM but $[f(z+c)P(f(z+c))]^{(2)} \not\equiv \frac{Q_2}{Q_1}[f(z)P(f(z))]$ as the condition $n \geqslant m+k+1$ is not satisfied.

2. Lemmas

LEMMA 1. [2] *Let $f \in \mathscr{F}$ be of finite order and c be non zero complex constant then*

$$m\left(r, \frac{f(z+c)}{f(z)}\right) + m\left(r, \frac{f(z)}{f(z+c)}\right) = S(r,f), \tag{2}$$

$$T(r, f(z+c)) = T(r,f) + S(r,f). \tag{3}$$

LEMMA 2. [8] *Let $f \in \mathscr{F}$ be of finite order and c be non zero complex constant. Let $P(z,f)$ be a polynomial in $f(z+c)$ and its derivatives and $Q(z,f)$ be a polynomial in $f(z), f(z+c)$ and its derivatives with meromorphic coefficients $a_\lambda, \lambda \in \mathbb{Z}$ such that $m(r,a_\lambda) = S(r,f)$. If $f^n(z)P(z,f) = Q(z,f)$ and the total degree of $Q(z,f)$ is n then*

$$m(r, P(z,f)) = S(r,f).$$

3. Proof of Theorem

Let

$$\begin{aligned} F &= f^n(z)P(f(z)), \\ G &= [f^n(z+c)P(f(z+c))]^{(k)}. \end{aligned} \tag{4}$$

Set

$$F_1(z) = \frac{F}{Q_1(z)} \quad \text{and} \quad G_1(z) = \frac{G}{Q_2(z)}. \tag{5}$$

Excluding the zeros of $Q_i(z)$, $i = 1,2$, $F_1(z)$ and $G_1(z)$ clearly share $(1,1)$ hence $\overline{N}\left(r, \frac{1}{F_1 - 1}\right) = \overline{N}\left(r, \frac{1}{G_1 - 1}\right) + S(r,f)$. Using (3) of lemma 1, we see that $S(r, f(z+c)) = S(r,f)$ and hence by (2) of lemma 1, we conclude that $m(r, \frac{G}{F}) = S(r,f)$. Define

$$\phi = \frac{F_1'(F_1 - G_1)}{F_1(F_1 - 1)}. \tag{6}$$

Case 1: Suppose $\phi \not\equiv 0$. Evidently $m(r,\phi) = S(r,f)$. Assume z_0 to be the zero of $f(z)$ with multiplicity $p(\geqslant 1)$ and zero of $P(f(z))$ with multiplicity $q(\geqslant 1)$ except for the zeros of $Q_i(z)$, $i = 1,2$. As $f(z)$ and $f(z+c)$ share 0 CM, we see that z_0 will be a zero of $f(z+c)$ with multiplicity p and zero of $P(f(z+c))$ with multiplicity q. From (5), z_0 will be zero of F_1 and G_1 with multiplicities $np+q$ and $np+q-k$ respectively. In this backdrop, (6) can be written as

$$\phi(z) = O((z - z_0)^{np+q-k-1}). \tag{7}$$

As $n \geqslant m+k+1$, it can be seen that $\phi(z)$ is holomorphic at z_0. Now lets say z_1 is a zero of $F_1 - 1$ with multiplicity $q_1(\geqslant 2)$ except for the zeros of $P(f)$ and $Q_i(z)$, $i = 1,2$. As F_1 and G_1 share $(1,1)$, we see that z_1 is a zero of $G_1 - 1$ with multiplicity

$r_1(\geqslant 2)$. In the neighbourhood of z_1, the Taylor series expansion of functions will be as follows:

$$F_1(z)-1=a_{q_1}(z-z_1)^{q_1}+a_{q_1+1}(z-z_1)^{q_1+1}+\dots,a_{q_1}\neq 0,$$
$$G_1(z)-1=b_{r_1}(z-z_1)^{r_1}+b_{r_1+1}(z-z_1)^{r_1+1}+\dots,b_{r_1}\neq 0,$$

$$F_1(z)-G_1(z)=\begin{cases} a_{q_1}(z-z_1)^{q_1}+a_{q_1+1}(z-z_1)^{q_1+1}+\dots if \quad q_1<r_1\\ -b_{r_1}(z-z_1)^{r_1}-b_{r_1+1}(z-z_1)^{r_1+1}\dots if \quad q_1>r_1\\ (a_{q_1}-b_{q_1})(z-z_1)^{q_1}+(a_{q_1+1}-b_{q_1+1})(z-z_1)^{q_1+1}\dots\text{if} \quad q_1=r_1. \end{cases}$$

$$F_1'(z)=q_1a_{q_1}(z-z_1)^{q_1-1}+(q_1+1)a_{q_1+1}(z-z_1)^{q_1}+\dots.$$

Let $t_1\geqslant\min\{q_1,r_1\}\geqslant 2$. With this regard, (6) can be rewritten as

$$\phi(z)=O((z-z_1)^{t_1-1}). \tag{8}$$

Clearly $\phi(z)$ is holomorphic at z_1. The zeros of $Q_i(z)$, $i=1,2$ and poles of $f(z)$ forms the poles of $\phi(z)$ which implies that $\phi(z)$ has finitely many poles hence $N(r,\phi)=O(\log r)$ in turn $T(r,\phi)=S(r,f)$. From (8), we see that

$$\overline{N}_{(2}\left(r,\frac{1}{F_1-1}\right)\leqslant N\left(r,\frac{1}{\phi}\right)\leqslant T(r,\phi)+S(r,f),$$
$$\Rightarrow\overline{N}_{(2}\left(r,\frac{1}{F_1-1}\right)=S(r,f).$$

Again as F_1 and G_1 share $(1,1)$ except for the zeros of $Q_i(z)$, $i=1,2$, we see that $\overline{N}_{(2}\left(r,\frac{1}{G_1-1}\right)=S(r,f)$. Rearranging the terms in (6), we get

$$\frac{1}{F_1}=\frac{F_1'}{\phi F_1(F_1-1)}\left(1-\frac{G_1}{F_1}\right). \tag{9}$$

From (5), we get $\frac{G_1}{F_1}=\frac{Q_1\,G}{Q_2\,F}$. Hence (9) will be of the form

$$\frac{1}{F_1}=\frac{F_1'}{\phi F_1(F_1-1)}\left(1-\frac{Q_1\,G}{Q_2\,F}\right).$$

Hence

$$m\left(r,\frac{1}{F_1}\right)=S(r,f)\quad\text{and}\quad m\left(r,\frac{1}{f}\right)=S(r,f). \tag{10}$$

Case 1.1: Suppose $n>m+k+1$. With reference to (7), we see that

$$N\left(r,\frac{1}{f}\right)\leqslant N\left(r,\frac{1}{\phi}\right)\leqslant T\left(r,\frac{1}{\phi}\right)\leqslant T(r,\phi)+S(r,f). \tag{11}$$

Combining (10) and (11) we get,

$$m\left(r,\frac{1}{f}\right)+N\left(r,\frac{1}{f}\right)=S(r,f),$$
$$\Rightarrow T(r,f)=S(r,f), \tag{12}$$

which proves contradiction.

Case 1.2: Suppose $n=m+k+1$ with reference to (7), we write

$$N_{(2}\left(r,\frac{1}{f}\right)\leqslant N\left(r,\frac{1}{\phi}\right)\leqslant T(r,\phi)+S(r,f), \quad \text{but}$$
$$T(r,f)=N\left(r,\frac{1}{f}\right)+m\left(r,\frac{1}{f}\right), \quad \text{hence}$$
$$T(r,f)=N_{1)}\left(r,\frac{1}{f}\right)+S(r,f). \tag{13}$$

By the definition of F and G, it is evident that

$$\overline{N}_{(2}\left(r,\frac{1}{F-Q_1}\right)=S(r,f) \quad \text{and} \quad \overline{N}_{(2}\left(r,\frac{1}{G-Q_2}\right)=S(r,f). \tag{14}$$

As $F-Q_1$ and $G-Q_2$ share $(0,1)$ there exists a meromorphic function say Γ of finite order such that

$$\frac{G-Q_2}{F-Q_1}=\Gamma \quad \text{(or)} \quad G-Q_2=\Gamma(F-Q_1). \tag{15}$$

Case 1.2.1: Now we consider the case when Γ is non constant. Let z_2 be a zero of Γ. As $F-Q_1$ and $G-Q_2$ share $(0,1)$ it clearly means that z_2 is a zero of $F-Q_1$ with multiplicity say p_2 and z_2 is a zero of $G-Q_2$ with multiplicity say q_2 such that $p_2<q_2$. In case if $p_2>q_2$ then z_2 becomes a pole of Γ. Since the poles of F and G are finite it is evident that $N(r,F)=S(r,f)$ and $N(r,G)=S(r,f)$. Therefore using (14) and (15) we can write

$$\overline{N}\left(r,\frac{1}{\Gamma}\right)\leqslant\overline{N}_{(2}\left(r,\frac{1}{G-Q_2}\right)=S(r,f),$$
$$\overline{N}(r,\Gamma)\leqslant\overline{N}_{(2}\left(r,\frac{1}{F-Q_1}\right)=S(r,f),$$

Differentiating (15), we get

$$G'-Q_2'=\Gamma'(F-Q_1)+\Gamma(F'-Q_1'). \tag{16}$$

In (16), replacing the term $F-Q_1$ and F from (15) and then rearranging, we arrive at

$$\begin{aligned} G'F-\frac{\Gamma'}{\Gamma}GF-GF' &= Q_1G-\left[\frac{\Gamma'}{\Gamma}Q_1+Q_1'\right]G-Q_2F'+\left[Q_2'-\frac{\Gamma'}{\Gamma}Q_2\right]F \\ &\quad +\frac{\Gamma'}{\Gamma}Q_1Q_2+Q_2Q_1'-Q_1Q_2'. \end{aligned} \tag{17}$$

Denote $\beta = \frac{\Gamma'}{\Gamma}$ and consequently $T(r,\beta) = S(r,f)$. $f(z)$ has finitely many poles and in addition $f(z)$ and $f(z+c)$ share 0 CM. Hence

$$f(z) = f(z+c)\ \psi(z)\ e^{\gamma(z)} \quad \text{(or)} \quad \frac{f(z)}{f(z+c)} = \psi(z)\ e^{\gamma(z)}, \tag{18}$$

where $\psi(z)$ is a rational function and $\gamma(z)$ is a polynomial. Differentiating (18), we get

$$f'(z) = f'(z+c)\ \psi(z)\ e^{\gamma(z)} + f(z+c)\ \psi'(z)\ e^{\gamma(z)} + f(z+c)\ \psi(z)\ e^{\gamma(z)}\ \gamma'(z). \tag{19}$$

Dividing (19) by (18), we get

$$\frac{f'(z)}{f(z)} = \frac{f'(z+c)}{f(z+c)} + \frac{\psi'(z)}{\psi(z)} + \gamma'(z).$$
$$m(r, \psi\ e^{\gamma}) = m\left(r, \frac{f(z)}{f(z+c)}\right).$$

From lemma 2, we conclude that

$$m(r, \psi\ e^{\gamma}) = S(r,f) \quad \text{and} \quad T(r, \psi\ e^{\gamma}) = S(r,f).$$

From (4), we have

$$\begin{aligned} G(z) &= \{a_m\ f^{m+n}(z+c) + a_{m-1}\ f^{m+n-1}(z+c) + \ldots + a_1\ f^{n+1}(z+c) \\ &\qquad + a_0\ f^n(z+c)\}^{(k)}, \\ &= \{G_m + G_{m-1} + \ldots + G_1 + G_0\}^{(k)} \\ &= \left\{\sum_{j=0}^{m} G_j\right\}^{(k)}, \text{ where } G_j = a_i\ f^{n+j}(z+c). \end{aligned} \tag{20}$$

For $k = 1$ differentiating the above equation,

$$G_i' = \sum_{i=0}^{m} a_i\ (n+i)\ f^{n+i-1}(z+c)\ f'(z+c),$$

For $k = 2$ differentiating the above equation,

$$\begin{aligned} G_i'' = \sum_{i=0}^{m} \big[&a_i\ (n+i)(n+i-1)\ f^{n+i-2}(z+c)\ [f'(z+c)]^2 \\ &+ a_i\ (n+i)\ f^{n+i-1}(z+c)\ f''(z+c)\big], \end{aligned}$$

For $k = 3$ differentiating the above equation,

$$\begin{aligned} G_i''' = \sum_{i=0}^{m} \big[&a_i\ (n+i)(n+i-1)\ (n+i-2)\ f^{n+i-3}(z+c)\ [f'(z+c)]^3 + 3a_i\ (n+i) \\ &(n+i-1)\ f^{n+i-2}(z+c)\ f'(z+c)\ f''(z+c) + a_i\ (n+i)\ f^{n+i-1}(z+c)\ f'''(z+c)\big]. \end{aligned}$$

In general differentiating k times we get,

$$G_i^{(k)} = \sum_{\lambda^i} a_{\lambda^i} \,(f(z+c))^{s_0^{\lambda^i}} \,(f'(z+c))^{s_1^{\lambda^i}} \,(f''(z+c))^{s_2^{\lambda^i}} \,\ldots\, (f^{(k)}(z+c))^{s_k^{\lambda^i}}, \tag{21}$$

where $s_0^{\lambda^i}, s_1^{\lambda^i}, \ldots, s_k^{\lambda^i} \in \mathbb{Z}^+$ such that $\sum_{j=0}^{k} s_j^{\lambda^i} = n+i$ and $n+i-k \leqslant s_0^{\lambda^i} \leqslant n+i-1$ for $i \in \{0,1,\ldots,m\}$ and a_{λ^i} are constants. Substituting (21) in (20), we get

$$\begin{aligned} G(z) = &\sum_{\lambda^m} a_{\lambda^m} \,(f(z+c))^{s_0^{\lambda^m}} \,(f')(z+c)^{s_1^{\lambda^m}} \,(f''(z+c))^{s_2^{\lambda^m}} \,\ldots\, (f^{(k)}(z+c))^{s_k^{\lambda^m}} + \\ &\sum_{\lambda^{m-1}} a_{\lambda^{m-1}} \,(f(z+c))^{s_0^{\lambda^{m-1}}} \,(f'(z+c))^{s_1^{\lambda^{m-1}}} \,(f''(z+c))^{s_2^{\lambda^{m-1}}} \,\ldots\, (f^{(k)}(z+c))^{s_k^{\lambda^{m-1}}} \\ &+\ldots+\sum_{\lambda^1} a_{\lambda^1} \,(f(z+c))^{s_0^{\lambda^1}} \,(f'(z+c))^{s_1^{\lambda^1}} \,(f''(z+c))^{s_2^{\lambda^1}} \,\ldots\, (f^{(k)}(z+c))^{s_k^{\lambda^1}} \\ &+\sum_{\lambda^0} a_{\lambda^0} \,(f(z+c))^{s_0^{\lambda^0}} \,(f'(z+c))^{s_1^{\lambda^0}} \,(f''(z+c))^{s_2^{\lambda^0}} \,\ldots\, (f^{(k)}(z+c))^{s_k^{\lambda^0}}. \end{aligned} \tag{22}$$

Differentiating (22), we get

$$\begin{aligned} G'(z) = &\sum_{\lambda^m} b_{\lambda^m} \,(f(z+c))^{t_0^{\lambda^m}} \,(f'(z+c))^{t_1^{\lambda^m}} \,(f''(z+c))^{t_2^{\lambda^m}} \,\ldots\, (f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^m}} + \\ &\sum_{\lambda^{m-1}} b_{\lambda^{m-1}} \,(f(z+c))^{t_0^{\lambda^{m-1}}} \,(f'(z+c))^{t_1^{\lambda^{m-1}}} \,(f''(z+c))^{t_2^{\lambda^{m-1}}} \,\ldots\, (f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^{m-1}}} \\ &+\ldots+\sum_{\lambda^1} b_{\lambda^1} \,(f(z+c))^{t_0^{\lambda^1}} \,(f'(z+c))^{t_1^{\lambda^1}} \,(f''(z+c))^{t_2^{\lambda^1}} \,\ldots\, (f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^1}} \\ &+\sum_{\lambda^0} b_{\lambda^0} \,(f(z+c))^{t_0^{\lambda^0}} \,(f'(z+c))^{t_1^{\lambda^0}} \,(f''(z+c))^{t_2^{\lambda^0}} \,\ldots\, (f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^0}}, \end{aligned} \tag{23}$$

where $t_0^{\lambda^i}, t_1^{\lambda^i}, \ldots, t_{k+1}^{\lambda^i} \in \mathbb{Z}^+$ such that $\sum_{j=0}^{k+1} t_j^{\lambda^i} = n+i$ and $n+i-k-1 \leqslant t_0^{\lambda^i} \leqslant n+i-1$ for $i \in \{0,1,\ldots,m\}$ and b_{λ^i} are constants. From (4), we have

$$F(z) = f^{m+n}(z)\left[a_m + \frac{a_{m-1}}{f(z)} + \ldots + \frac{a_1}{f^{m-1}(z)} + \frac{a_0}{f^m(z)}\right]. \tag{24}$$

Differentiating (24), we get

$$F'(z) = f^{m+n}(z)\,f'(z)\left[\frac{a_m(m+n)}{f(z)} + \frac{a_{m-1}(m+n-1)}{f^2(z)} + \ldots + \frac{a_1(n+1)}{f^m(z)} + \frac{a_0\,n}{f^{m+1}(z)}\right]. \tag{25}$$

Substituting (22), (23), (24) and (25) in (17), we see that

$$
\begin{aligned}
f^{m+n}(z)\Bigg\{G'\left[a_m+\frac{a_{m-1}}{f(z)}+\ldots+\frac{a_1}{f^{m-1}(z)}+\frac{a_0}{f^m(z)}\right]-\beta\, G\left[a_m+\frac{a_{m-1}}{f(z)}+\ldots\right.\\
\left.+\frac{a_1}{f^{m-1}(z)}+\frac{a_0}{f^m(z)}\right]-G\frac{f'(z)}{f(z)}\left[a_m\,(m+n)+\frac{a_{m-1}(m+n-1)}{f(z)}+\ldots\right.\\
\left.+\frac{a_1(n+1)}{f^{m-1}(z)}+\frac{a_0\,n}{f^m(z)}\right]\Bigg\}=Q(z),
\end{aligned}
\tag{26}
$$

where $Q(z)$ is a differential polynomial in $f(z)$ and $f(z+c)$ of degree n. Let

$$
\begin{aligned}
P_1(z)&=a_m+\frac{a_{m-1}}{f(z)}+\ldots+\frac{a_1}{f^{m-1}(z)}+\frac{a_0}{f^m(z)},\\
P_2(z)&=a_m\,(m+n)+\frac{a_{m-1}(m+n-1)}{f(z)}+\ldots+\frac{a_1(n+1)}{f^{m-1}(z)}+\frac{a_0\,n}{f^m(z)},\\
&G'(z)\,P_1(z)-\beta\,G(z)\,P_1(z)-G(z)\,\frac{f'(z)}{f(z)}\,P_2(z)=P(z).
\end{aligned}
\tag{27}
$$

Using (27) in (26), we get

$$
f^{m+n}(z)\,P(z)=Q(z). \tag{28}
$$

From (20), $\frac{f'(z)}{f(z)}$ can be replaced by $\frac{f'(z+c)}{f(z+c)}+\frac{\psi'(z)}{\psi(z)}+\gamma'(z)$. Hence from (27),

$$
\begin{aligned}
P(z)=P_1(z)\Bigg\{&\sum_{\lambda^m}b_{\lambda^m}\,(f(z+c))^{t_0^{\lambda^m}}\,(f'(z+c))^{t_1^{\lambda^m}}\,\ldots\,(f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^m}}\\
&+\sum_{\lambda^{m-1}}b_{\lambda^{m-1}}\,(f(z+c))^{t_0^{\lambda^{m-1}}}\,(f'(z+c))^{t_1^{\lambda^{m-1}}}\,\ldots\,(f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^{m-1}}}\\
&+\ldots+\sum_{\lambda^1}b_{\lambda^1}\,(f(z+c))^{t_0^{\lambda^1}}\,(f'(z+c))^{t_1^{\lambda^1}}\,\ldots\,(f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^1}}\\
&+\sum_{\lambda^0}b_{\lambda^0}\,(f(z+c))^{t_0^{\lambda^0}}\,(f'(z+c))^{t_1^{\lambda^0}}\,\ldots\,(f^{(k+1)}(z+c))^{t_{k+1}^{\lambda^0}}\\
&-\beta\Bigg\{\sum_{\lambda^m}a_{\lambda^m}\,(f(z+c))^{s_0^{\lambda^m}}\,(f'(z+c))^{s_1^{\lambda^m}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^m}}\\
&+\sum_{\lambda^{m-1}}a_{\lambda^{m-1}}\,(f(z+c))^{s_0^{\lambda^{m-1}}}\,(f'(z+c))^{s_1^{\lambda^{m-1}}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^{m-1}}}+\ldots\\
&+\sum_{\lambda^1}a_{\lambda^1}\,(f(z+c))^{s_0^{\lambda^1}}\,(f'(z+c))^{s_1^{\lambda^1}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^1}}\\
&+\sum_{\lambda^0}a_{\lambda^0}\,(f(z+c))^{s_0^{\lambda^0}}\,(f'(z+c))^{s_1^{\lambda^0}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^0}}\Bigg\}\Bigg\}
\end{aligned}
$$

$$-f'(z+c)\,P_2(z)\left\{\sum_{\lambda^m} a_{\lambda^m}(f(z+c))^{s_0^{\lambda^m}-1}\,(f'(z+c))^{s_1^{\lambda^m}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^m}}\right.$$

$$+\sum_{\lambda^{m-1}} a_{\lambda^{m-1}}(f(z+c))^{s_0^{\lambda^{m-1}}-1}\,(f'(z+c))^{s_1^{\lambda^{m-1}}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^{m-1}}}+\ldots$$

$$+\sum_{\lambda^1} a_{\lambda_1}(f(z+c))^{s_0^{\lambda^1}-1}\,(f'(z+c))^{s_1^{\lambda^1}}\,\ldots\,(f^{(k)}(z+c))^{s_k^{\lambda^1}}$$

$$\left.+\sum_{\lambda^0} a_{\lambda_0}(f(z+c))^{s_0^{\lambda^0}-1}\,(f'(z+c))^{s_1^{\lambda^0}}\,\ldots\,(f(z+c)^{(k)})^{s_k^{\lambda^0}}\right\}$$

$$-G\left[\frac{\psi'(z)}{\psi(z)}+\gamma'(z)\right]\,P_2(z).$$

The term $G\left[\frac{\psi'(z)}{\psi(z)}+\gamma'(z)\right]\,P_2(z)$ in the above equation does not contain highest power of $f'(z+c)$ and hence can be neglected. In general $P(z)$ will be a differential polynomial in $f(z+c)$ of degree $k+1$ which can be written in the following form

$$P(z)=H\,[f'(z+c)]^{k+1}+I_*(f), \tag{29}$$

where H is a suitable constant and $I_*(f)$ is a polynomial. In specific $I_*(f)$ is of the form

$$I_*(f)=S(\beta,\,\beta',\,\psi,\,\psi',\,\gamma')\left\{\sum_{\lambda^m}(f(z+c))^{u_0^{\lambda^m}}\,(f'(z+c))^{u_1^{\lambda^m}}\ldots(f^{(k+1)}(z+c))^{u_{k+1}^{\lambda^m}}\right.$$

$$+\sum_{\lambda^{m-1}}(f(z+c))^{u_0^{\lambda^{m-1}}}\,(f'(z+c))^{u_1^{\lambda^{m-1}}}\ldots(f^{(k+1)}(z+c))^{u_{k+1}^{\lambda^{m-1}}}+\ldots+$$

$$\sum_{\lambda^1}(f(z+c))^{u_0^{\lambda^1}}\,(f'(z+c))^{u_1^{\lambda^1}}\ldots(f^{(k+1)}(z+c))^{u_{k+1}^{\lambda^1}}$$

$$\left.+\sum_{\lambda^0}(f(z+c))^{u_0^{\lambda^0}}\,(f'(z+c))^{u_1^{\lambda^0}}\ldots(f^{(k+1)}(z+c))^{u_{k+1}^{\lambda^0}}\right\},$$

where $u_0^{\lambda^i},u_1^{\lambda^i},\ldots,u_{k+1}^{\lambda^i}\in\mathbb{Z}^+$ such that $\sum_{j=0}^{k+1}u_j^{\lambda^i}=n+2i$ and $n+2i-k\leqslant u_0^{\lambda^i}\leqslant n+2i-1$ for $i\in\{0,1,\ldots,m\}$ and $S(\beta,\,\beta',\,\psi,\,\psi',\,\gamma')$ is a polynomial in $\beta,\,\beta',\,\psi,\,\psi',\,\gamma'$ with constant coefficients. With reference to (28), we take up two cases:

Case 1.2.1.1: Suppose $P(z)\not\equiv 0$. Using lemma 2, we see that $m(r,P)=S(r,f)$ and hence

$$T(r,P)=S(r,f)\quad\text{and}\quad T(r,P')=S(r,f). \tag{30}$$

Differentiating (29), we get

$$P'(z)=H\,(k+1)\,[f'(z+c)]^k\,f''(z+c)+L\,S(z)\,[f'(z+c)]^{k+1}+S_1(z), \tag{31}$$

where L is a suitable constant, $S(z)=S(\beta,\ \beta',\ \psi,\ \psi',\ \gamma')$ and $S_1(z)$ is a polynomial of the form

$$\begin{aligned}S_1(z)=S(z)\ \Bigg\{&\sum_{\lambda^m}(f(z+c))^{v_0^{\lambda^m}}\ (f'(z+c))^{v_1^{\lambda^m}}\ \ldots\ (f^{(k+1)}(z+c))^{v_{k+1}^{\lambda^m}}\\&+\sum_{\lambda^{m-1}}(f(z+c))^{v_0^{\lambda^{m-1}}}\ (f'(z+c))^{v_1^{\lambda^{m-1}}}\ \ldots\ (f^{(k+1)}(z+c))^{v_{k+1}^{\lambda^{m-1}}}\\&+\ldots+\sum_{\lambda^1}(f(z+c))^{v_0^{\lambda^1}}\ (f'(z+c))^{v_1^{\lambda^1}}\ \ldots\ (f^{(k+1)}(z+c))^{v_{k+1}^{\lambda^1}}\\&+\sum_{\lambda^0}(f(z+c))^{v_0^{\lambda^0}}\ (f'(z+c))^{v_1^{\lambda^0}}\ \ldots\ (f^{(k+1)}(z+c))^{v_{k+1}^{\lambda^0}}\Bigg\},\end{aligned}$$

where $v_0^{\lambda^i},v_1^{\lambda^i},\ldots,v_{k+1}^{\lambda^i}\in\mathbb{Z}^+$ such that $\sum\limits_{j=0}^{k+1}v_j^{\lambda^i}=n+2i$ and $n+2i-k\leqslant v_0^{\lambda^i}\leqslant n+2i-1$ for $i\in\{0,1,\ldots,m\}$. Assume z_3 to be a simple zero of $f(z+c)$ except for the zeros of Γ and Γ'. So (29) and (31) can be written as

$$P(z_3)=H\ [f'(z_3+c)]^{k+1}, \tag{32}$$

$$P'(z_3)=H\ (k+1)[f'(z_3+c)]^k\ f''(z_3+c)+L\,S(z_3)\ [f'(z_3+c)]^{k+1}. \tag{33}$$

Using (32) in (33) and then rearranging, we get

$$P(z_3)\ f''(z_3+c)-\frac{P'(z_3)\ f'(z_3)}{k+1}+\frac{L\,S(z_3)\ P(z_3)\ f'(z_3+c)}{H\ (k+1)}=0. \tag{34}$$

Let $K_1(z)=\dfrac{1}{k+1}$ and $K_2(z)=\dfrac{L\,S(z_3)}{H\ (k+1)}$, so (34) becomes

$$P(z_3)\ f''(z_3+c)-[K_1(z)\ P'(z_3)-K_2(z)\ P(z_3)]f'(z_3+c)=0.$$

Clearly z_3 is a zero of $P(z)\ f''(z+c)-[K_1(z)\ P'(z)-K_2(z)\ P(z)]f'(z+c)$ and consequently $T(r,K_1)=S(r,f)$ and $T(r,K_2)=S(r,f)$. Lets define

$$\phi_1(z)=\frac{P(z)\ f''(z+c)-[K_1(z)\ P'(z)-K_2(z)\ P(z)]f'(z+c)}{f(z+c)}. \tag{35}$$

Let

$$v_1(z)=\frac{\phi_1(z)}{P(z)}\quad\text{and}\quad v_2(z)=\frac{K_1(z)\ P'(z)}{P(z)}-K_2(z). \tag{36}$$

Using (36), (35) can be written as

$$f''(z+c)=v_1(z)\ f(z+c)+v_2(z)\ f'(z+c). \tag{37}$$

Clearly $T(r,\nu_1)=S(r,f)$ and $T(r,\nu_2)=S(r,f)$. Suppose if $\phi_1(z)\equiv 0$ then $\nu_1(z)=0$; the detailed analysis of this case is on the same lines of the equation (3.24) in [12]. Lets suppose if $\phi_1(z)\not\equiv 0$, then from (36), we have

$$P'(z)=\left[\frac{\nu_2(z)}{K_1(z)}+\frac{K_2(z)}{K_1(z)}\right]P(z). \tag{38}$$

Substituting (29) in (38), we get

$$P'(z)=\left[\frac{\nu_2(z)}{K_1(z)}+\frac{K_2(z)}{K_1(z)}\right]H\,[f'(z+c)]^{k+1}+\left[\frac{\nu_2(z)}{K_1(z)}+\frac{K_2(z)}{K_1(z)}\right]I_*(f). \tag{39}$$

Substituting (37) in (31), we get

$$\begin{aligned}P'(z)=H\,(k+1)\,\nu_1(z)\,[f'(z+c)]^k\,f(z+c)+[H\,(k+1)\,\nu_2(z)+L\,S(z)]\\ [f'(z+c)]^{k+1}+S_1(z).\end{aligned} \tag{40}$$

Comparing equations (39) and (40), we see that

$$\begin{aligned}&\left[H\left(\frac{\nu_2(z)}{K_1(z)}+\frac{K_2(z)}{K_1(z)}\right)-H\,(k+1)\,\nu_2(z)-L\,S(z)\right][f'(z+c)]^{k+1}\\ &-H\,(k+1)\,\nu_1(z)\left[f'(z+c)\right]^k\,f(z+c)+\left[\frac{\nu_2(z)}{K_1(z)}+\frac{K_2(z)}{K_1(z)}\right]I_*(f)-S_1(z)\equiv 0.\end{aligned}$$

Since $\nu_1(z)\not\equiv 0$, from (41), we have

$$N_{1)}\left(r,\frac{1}{f}\right)=S(r,f). \tag{41}$$

Using equations (13) and (41), we see that $T(r,f)=S(r,f)$ which is a contradiction.

Case 1.2.1.2: Suppose $P(z)\equiv 0$. From (28), we see that $Q(z)\equiv 0$ hence (17) becomes

$$G'\,F-\frac{\Gamma'}{\Gamma}\,G\,F-G\,F'\equiv 0 \quad (\text{or}) \quad \frac{G'}{G}=\frac{\Gamma'}{\Gamma}+\frac{F'}{F}. \tag{42}$$

On Integrating (42), we get $G=d\,\Gamma F$ where d is a non zero constant. We have $n=m+k+1$ and $\overline{N}(r,\Gamma)=S(r,f)$ hence from (15) it follows that $\overline{N}\left(r,\frac{1}{f}\right)=S(r,f)$ and consequently from (13), $T(r,f)=S(r,f)$ which leads to a contradiction.

Case 1.2.2: Let us consider the case when Γ is a constant say D such that $D\neq 0$. From (15), we can write

$$G-Q_2=D(F-Q_1) \quad (\text{or}) \quad G-DF=Q_2-DQ_1. \tag{43}$$

We have $n=m+k+1$, it follows that $\overline{N}\left(r,\frac{1}{f}\right)=S(r,f)$ and consequently from (13), $T(r,f)=S(r,f)$ which leads to a contradiction.

Case 2: Suppose $\phi \equiv 0$. From (6), we get $F_1 \equiv G_1$ i.e.,

$$[f^n(z+c)P(f(z+c))]^{(k)} \equiv \frac{Q_2}{Q_1}[f^n(z)P(f(z))]. \tag{44}$$

Furthermore if $Q_1 \equiv Q_2$, then

$$[f^n(z+c)P(f(z+c))]^{(k)} \equiv [f^n(z)P(f(z))]. \tag{45}$$

Lets assume z_4 to be the zero of $f(z)$ with multiplicity say r. As $f(z)$ and $f(z+c)$ share 0 CM, z_4 will be a zero of $f(z+c)$ with multiplicity t, z_4 is a zero of $[f^n(z)P(f(z))]$ with multiplicity $(m+n)r$ and z_4 is a zero of $[f^n(z+c)P(f(z+c))]^{(k)}$ with multiplicity $(m+n)r-k$. This will be a contradiction in the backdrop of (45). Denote:

$$G_2(z) = f^n(z+c)P(f(z+c)) = a_i f^{n+i}(z+c),\ i \in \{0,1,\dots m\}. \tag{46}$$

Clearly from (46), $[G_2(z)]^{(k)} \neq 0$ as $f(z) \neq 0$ and $f(z+c) \neq 0$. Now as $f(z)$ is a transcendental meromorphic function with finitely many poles and $f(z) \neq 0$, $f(z)$ must be of the form

$$f(z) = \frac{e^{L_2(z)}}{L_1(z)}, \tag{47}$$

where $L_1(z)$ is a non zero polynomial and $L_2(z)$ is a non constant polynomial. Using (47) in (46), $G_2(z)$ can be re-written as

$$G_2(z) = a_i \frac{e^{L_4(z)}}{L_3(z)}, \quad \text{where} \quad L_3(z) = L_1^{n+i}(z+c),\ L_4(z) = (n+i)L_2(z+c). \tag{48}$$

Define:

$$\chi(z) = \frac{G_2'(z)}{G_2(z)} = L_4'(z) - \frac{L_3'(z)}{L_3(z)}. \tag{49}$$

Using (49) in lemma 2.4 of [12], we get

$$\frac{G_2^{(k)}(z)}{G_2(z)} = \chi^k + Q_{k-1}(\chi), \tag{50}$$

where $Q_{k-1}(\chi)$ is a polynomial of degree $k-1$ in χ and its derivative.

Case 2.1: Suppose L_4' is not a constant. Clearly $\frac{G_2^{(k)}(z)}{G_2(z)} \sim \chi^k \sim (L_4')^k \to \infty$ as $z \to \infty$. We know that every non-constant rational function assumes every value in the closed complex plane. Hence $G_2(z)^{(k)} \equiv 0$ somewhere in the open complex plane which is a contradiction.

Case 2.2: Suppose L_4' is a constant. Let Ł$_4' = \lambda \neq 0$. Suppose if $\chi(z)$ is non constant, then from (49), we have $\chi(z) = \lambda$ and consequently $\chi'(z) = \chi''(z) = \dots = \chi^{(k)}(z) = 0$ as $z \to \infty$. Using lemma 2.4 of [12], $\frac{G_2^{(k)}(z)}{G_2(z)} = \lambda^k, z \to \infty$. This means that $\dfrac{G_2^{(k)}(z)}{G_2(z)}$ must have a zero in the open complex plane and incidentally $\chi(z)$ is a constant. Therefore $L_4' = \lambda = \chi(z)$. From (50) we get,

$$G_2(z) = e^{\lambda z + d}, \tag{51}$$

where d is a constant and consequently from (46) we get,

$$f(z) = C\, e^{\left(\dfrac{\lambda z}{n+i}\right)}, \tag{52}$$

where $C(\neq 0) = \dfrac{c_*}{a_i}$ is a constant such that $e^{\lambda C} = 1$, c_* is an integration constant and $\lambda^k = 1$. $\square$

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