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AN ALTERNATIVE APPROACH TO IDEAL WIJSMAN CONVERGENCE

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Abstract. The concept of Wijsman convergence of a sequence of sets was defined using the pointwise convergence of the sequence of distance functions. Based on this idea, in this article, a new type of set convergence is obtained by using the concept of ideal α-convergence for the sequence of distance functions. Then it is shown that this new convergence is equivalent to the Wijsman convergence according to the specific choice of ideal.

1. Introduction

The theory of ideal convergence has been introduced by Kostyrko et al. [9] in the first year of this century. In 2004, Salat et al. [18] extended some of the results given for monotone sequences in statistical convergence to ideal convergence. Then Kostyrko et al. [10] defined the $\mathscr{I}$-limit inferior and $\mathscr{I}$-limit superior of a sequence and obtained some results related to these concepts. The notion of rough $\mathscr{I}$-convergence were independently introduced by Dündar and Çakan [5] and Pal et al. [17].

Wijsman introduced the concept of pointwise convergence of sequences of distance functions, which has an important role in the theory of convergence of sets. Then this convergence type was called his name. Recently some authors have obtained some new convergence types applying the theory of Wijsman convergence to different areas. In this context, the concept of statistical Wijsman convergence was first introduced by Nuray and Rhoades [13]. Ölmez and Aytar [15] defined the idea of rough Wijsman convergence of a sequence of sets. Then, Subramanian and Esi [20] defined the concept of rough Wijsman convergence for a triple sequences of sets. On the other hand, the definition of $\mathscr{I}$-Wijsman convergence were given by Kişi and Nuray [8]. Then Nuray et al. [14] defined the concept of $\mathscr{I}_2$-Wijsman convergence for double sequences.

In [7] and [19], the idea of continuous convergence has been introduced for the sequences of functions. Later, this notion started to be referred to as α-convergence in the literature. Combining the theories of α-convergence and equal convergence, Das and Papanastassiou [4] defined some new convergence types and examined the relationships between them. Gregoriades and Papanastassiou [6] investigated the effect of the exhaustive concept on pointwise, uniform and α-convergence of sequences of functions. Athanassiadou et al. [2] stated the relationships among the concepts pointwise convergence, α-convergence and $\mathscr{I}$-pointwise convergence. Then Albayrak and Pehlivan [1] generalized these convergence types by using the filters on $\mathbb{N}$.

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In this article, we introduced the concept of ideal α-convergence for the sequences of sets. We have shown that this new definition is equivalent to the ideal Wijsman convergence by selecting the special ideal $\mathscr{I}_f$.

2. Preliminaries

The definitions introduced in this section are basic and will be used in the next section. Throughout this paper, let (X,ρ_X) be a metric space and A, A_n be nonempty closed subsets of X for each $n\in\mathbb{N}$.

The *distance function* $d(\cdot,A) : X \longrightarrow [0,\infty)$ is defined by the formula

$$d(x,A) = \inf\{\rho_X(x,y) : y \in A\}$$

[7, 21].

We say that the sequence (A_n) is *Wijsman convergent* to the set A if

$$\lim d(x,A_n) = d(x,A) \text{ for all } x \in X.$$

In this case, we write $A_n \xrightarrow{W} A$, as $n\to\infty$ [21, 22].

Let (Y,ρ_Y) be another metric space and D be a subset of X. Assume the f, f_n functions from X to Y for each $n\in\mathbb{N}$. The sequence (f_n) α*-converges* to f iff for every $x\in X$ and for every sequence (x_n) of points of X converging to x, the sequence $(f_n(x_n))$ converges to $f(x)$. We shall write $f_n \xrightarrow{\alpha} f$ to denote that the sequence (f_n) α-converges to f (see [6, 7, 19]).

The open ball with centre $x\in X$ and radius $\delta>0$ is the set

$$S(x,\delta) = \{y\in X : \rho_X(x,y) < \delta\}.$$

The sequence (f_n) is called *equicontinuous* at x if for all $\varepsilon>0$ there exists $\delta=\delta(\varepsilon)>0$ such that $\rho_Y(f_n(y),f_n(x))<\varepsilon$ whenever $y\in S(x,\delta)$, $n\in\mathbb{N}$ [7].

Briefly, we recall some of basic notations in the theory of $\mathscr{I}$-convergence, and we refer readers to [7, 9, 11] and the recent monographs [12] and [3], for more details and related topics.

A family $\mathscr{I}\subseteq 2^{\mathbb{N}}$ of subsets of $\mathbb{N}$ is said to be an *ideal* in $\mathbb{N}$ if $\emptyset\in\mathscr{I}$, and $A\cup B\in\mathscr{I}$ for each $A,B\in\mathscr{I}$, and $B\in\mathscr{I}$ for each $A\in\mathscr{I}$ such that $B\subseteq A$ [11].

An ideal is called *proper* if $\mathbb{N}\notin\mathscr{I}$, and proper ideal is called *admissible* if $\{n\}\in\mathscr{I}$ for each $n\in\mathbb{N}$. Obviously, an admissible ideal includes all finite subset of $\mathbb{N}$ [9]. In addition, ideal $\mathscr{I}$ is called P*-ideal* if for every sequence $(M_n)_{n\in\mathbb{N}}$ of sets from $\mathscr{I}$ there is a set $M\in\mathscr{I}$ such that $M_n\setminus M$ is finite for each $n\in\mathbb{N}$. A property of P-ideals: there exists a set $M\in\mathscr{I}$ such that the subsequence $(x_n)_{n\in M^c}$ is convergent to x for each $\mathscr{I}$-convergent sequence (x_n), where M^c is the complement of the set M, that is $M^c=\mathbb{N}\setminus M$.

Define $\mathscr{I}_f=\{A\subset\mathbb{N} : \text{the set } A \text{ has finite number of elements}\}$. Then $\mathscr{I}_f$-convergence and classical convergence is equivalent to each other. Similarly if we denote $\mathscr{I}_d=\{A\subset\mathbb{N} : \text{the set } A \text{ has natural density zero}\}$, then $\mathscr{I}_d$-convergence and statistical convergence is equivalent to each other. We note that the ideals $\mathscr{I}_f$ and $\mathscr{I}_d$ are admissible.

Let $\mathscr{I} \subset 2^{\mathbb{N}}$ be a proper ideal in $\mathbb{N}$. We shall say that the sequence (A_n) of sets $\mathscr{I}$-*Wijsman converges* to the set A if

$$\{n \in \mathbb{N} : |d(x,A_n) - d(x,A)| \geqslant \varepsilon\} \in \mathscr{I}$$

for every $\varepsilon > 0$ and each $x \in X$. That is, $\mathscr{I}\text{-}\lim d(x,A_n) = d(x,A)$ for each $x \in X$. In this case, we write $A_n \xrightarrow{\mathscr{I}\text{-}W} A$, as $n \to \infty$ [8].

It is clear that if $\mathscr{I}$ is an admissible ideal and a sequence (A_n) of sets is Wijsman convergent then it is $\mathscr{I}$-Wijsman convergent to the same set. It is showed by the following Example 1 that the converse of this statement does not generally hold.

EXAMPLE 1. Define

$$A_n := \begin{cases} \{(n,n)\} & , n = k^2 \\ \left\{\left(-\dfrac{1}{n}, -\dfrac{1}{n}\right)\right\} & , n \neq k^2 \end{cases} ; \; k \in \mathbb{N}$$

and $A = \{(0,0)\}$ in the space $\mathbb{R}^2$ equipped with the Euclidean metric. Let $(x^*,y^*) \in \mathbb{R}^2$. Then we have

$$\begin{aligned} &|d((x^*,y^*),A_n) - d((x^*,y^*),A)| \\ &= \begin{cases} \left|\sqrt{(x^*-n)^2 + (y^*-n)^2} - \sqrt{(x^*-0)^2 + (y^*-0)^2}\right| & , n = k^2 \\ \left|\sqrt{\left(x^* + \frac{1}{n}\right)^2 + \left(y^* + \frac{1}{n}\right)^2} - \sqrt{(x^*-0)^2 + (y^*-0)^2}\right| & , n \neq k^2 \end{cases} \end{aligned}$$

and for every $\varepsilon > 0$

$$\begin{aligned} &\{n \in \mathbb{N} : |d((x^*,y^*),A_n) - d((x^*,y^*),A)| \geqslant \varepsilon\} \\ &\subseteq \left\{1,4,9,\ldots,k^2,\ldots\right\} \cup \{1,2,\ldots,N(\varepsilon)\} = K(\varepsilon) \end{aligned}$$

where $N(\varepsilon) \in \mathbb{N}$ and $N(\varepsilon) > \dfrac{\sqrt{2}}{\varepsilon}$. Since $K(\varepsilon)$ has natural density zero, the sequence (A_n) is $\mathscr{I}_d$-Wijsman convergent to the set A.

However, since $\{n \in \mathbb{N} : |d((x^*,y^*),A_n) - d((x^*,y^*),A)| \geqslant \varepsilon\}$ is an infinite set we obtain $A_n \stackrel{W}{\not\longrightarrow} A$. This means that although there are $\mathscr{I}_d$-Wijsman convergent sequences but there are sequences that are not Wijsman convergent.

3. Ideal α-convergence

We begin our results by extending concept of Wijsman convergence.

DEFINITION 1. The sequence (A_n) is said to be $\mathscr{I}\text{-}\alpha$-convergent to the set A if for every $x \in X$ and every sequence (x_n) such that $x_n \longrightarrow x$, the condition $\mathscr{I} - \lim d(x_n,A_n) = d(x,A)$ holds. Then we write $A_n \xrightarrow{\mathscr{I}-\alpha} A$, as $n \to \infty$.

In Definition 1, ordinary convergence is used for the sequence (x_n). If we replace ordinary convergence with the ideal convergence, then we get the following definition.

DEFINITION 2. We shall say that a sequence (A_n) of sets $\mathscr{I}_1\mathscr{I}_2$-$\alpha$-converges to the set A, written $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\longrightarrow} A$, as $n\to\infty$, if for every $x\in X$ and every sequence (x_n) such that $\mathscr{I}_1-\lim x_n = x$, the condition $\mathscr{I}_2-\lim d(x_n,A_n) = d(x,A)$ holds.

If we take the ideal $\mathscr{I}_f$ as the ideal $\mathscr{I}_1$ in Definition 2, then we say that Definition 2 is a generalization of Definition 1.

EXAMPLE 2. Let $\mathscr{I}_2 = \mathscr{I}_d$ and $\mathscr{I}_1 = \{B\subseteq\mathbb{N} : \exists k\in\mathbb{N} \text{ such that } B\cap\mathbb{N}_k = \emptyset\}$, where $\mathbb{N}_k = \{nk : n\in\mathbb{N}\}$. Define

$$A_n = \begin{cases} \left[\frac{1}{n},4\right]\times[-2,2] & \text{, if } n\notin\mathbb{P} \\ [-n,-1]\times[-2,2] & \text{, if } n\in\mathbb{P} \end{cases}$$

($\mathbb{P}$ is the set of all prime numbers) and $A=[0,4]\times[-2,2]$ in the space $\mathbb{R}^2$ equipped with the Euclidean metric. Let $a\in A$ and define the sequence $(x_n)_{n\in\mathbb{N}}$ by

$$x_n = \begin{cases} (0,0) & \text{, if } n=2m \\ \left(-\frac{1}{2},0\right) & \text{, if } n=2m-1 \end{cases} ;m\in\mathbb{N}.$$

Then we have $\mathscr{I}_1-\lim x_n = (0,0) = x$. But, since

$$d(x_n,A_n) = \begin{cases} \frac{1}{n} & \text{, if } n=2m+2 \\ 1 & \text{, if } n=2 \\ \frac{1}{2}+\frac{1}{n} & \text{, if } n=2m-1,\ n\notin\mathbb{P} \\ \frac{1}{2} & \text{, if } n=2m-1,\ n\in\mathbb{P} \end{cases},$$

we obtain $\mathscr{I}_2-\lim d(x_n,A_n) \neq 0 = d(x,A)$.

Hence we have $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\not\longrightarrow} A$, but this sequence is $\mathscr{I}_2$-α-Wijsman convergent to the set A.

The following proposition shows the relation between Definitions 1 and 2.

PROPOSITION 1. Let $\mathscr{I}_1$ be an admissible ideal. Assume that the sequence (A_n) is $\mathscr{I}_1\mathscr{I}_2$-$\alpha$-convergent to the set A. Then the following hold:

a) (A_n) is $\mathscr{I}_2$-α-convergent to the set A.

b) If $\mathscr{I}_1\supseteq\mathscr{I}_2$ then (A_n) is $\mathscr{I}_1$-α-convergent to the set A.

Proof. Assume $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\longrightarrow} A$. Take $x\in X$. Let (x_n) be a sequence such that $x_n\longrightarrow x$. Since $\mathscr{I}_1$ is an admissible ideal, we also have $\mathscr{I}_1-\lim x_n = x$.

a) Since $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\longrightarrow} A$, we get

$$\mathscr{I}_2 - \lim d(x_n, A_n) = d(x, A). \tag{1}$$

Then (1) holds for each sequence (x_n) such that $x_n \longrightarrow x$. Hence we have $A_n \overset{\mathscr{I}_2-\alpha}{\longrightarrow} A$.

b) Since $\mathscr{I}_1 \supseteq \mathscr{I}_2$, we have

$$\mathscr{I}_2 - \lim d(x_n, A_n) = d(x, A) \Longrightarrow \mathscr{I}_1 - \lim d(x_n, A_n) = d(x, A)$$

from (1). Then we get $A_n \overset{\mathscr{I}_1-\alpha}{\longrightarrow} A$. $\square$

COROLLARY 1. *Let $\mathscr{I}$ be an admissible ideal and $\mathscr{I}_1 = \mathscr{I}_2 = \mathscr{I}$. If $A_n \overset{\mathscr{I}\mathscr{I}-\alpha}{\longrightarrow} A$ then $A_n \overset{\mathscr{I}-\alpha}{\longrightarrow} A$.*

The converse of Proposition 1 holds for the class of P-ideals.

PROPOSITION 2. Let $\mathscr{I}_1$ be an P-ideal and $\mathscr{I}_1 \subseteq \mathscr{I}_2$. If $A_n \overset{\mathscr{I}_2-\alpha}{\longrightarrow} A$, then $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\longrightarrow} A$.

Proof. Let $\varepsilon > 0$ and $x \in X$. Let us assume that $\mathscr{I}_1 - \lim x_n = x$. Since $\mathscr{I}_1$ is an P-ideal, there is an $M \in \mathscr{I}_1$ such that the subsequence $(x_n)_{n \in M^c}$ is convergent to x. Let us define the sequence $(y_n)_{n\in\mathbb{N}}$ by

$$y_n := \begin{cases} x & \text{, if } n \in M \\ x_n & \text{, otherwise} \end{cases}$$

for each $n \in \mathbb{N}$. It is clear that $y_n \longrightarrow x$. Since $A_n \overset{\mathscr{I}_2-\alpha}{\longrightarrow} A$, we have

$$K(\varepsilon) := \{n \in \mathbb{N} : |d(y_n, A_n) - d(x, A)| \geqslant \varepsilon\} \in \mathscr{I}_2.$$

Since $M \in \mathscr{I}_1$ and $\mathscr{I}_1 \subseteq \mathscr{I}_2$, we have $M \in \mathscr{I}_2$. We also have $M \cup K(\varepsilon) \in \mathscr{I}_2$. For every $n \in \mathbb{N} \setminus [M \cup K(\varepsilon)] = M^c \cap (\mathbb{N} \setminus K(\varepsilon))$, we get

$$|d(x_n, A_n) - d(x, A)| < \varepsilon.$$

Then we have

$$\{n \in \mathbb{N} : |d(x_n, A_n) - d(x, A)| \geqslant \varepsilon\} \subseteq M \cup K(\varepsilon) \in \mathscr{I}_2$$

and so

$$\{n \in \mathbb{N} : |d(x_n, A_n) - d(x, A)| \geqslant \varepsilon\} \in \mathscr{I}_2.$$

Consequently, we get $A_n \overset{\mathscr{I}_1\mathscr{I}_2-\alpha}{\longrightarrow} A$. $\square$

Now we give two lemmas which will be used in the proof of Theorem 1. The proof of Lemma 1 is clear from the Lipschitz continuity of the distance function and another Lemma 2's proof was given in [16].

LEMMA 1. *If the set A is a nonempty closed subset of X, then we have*

$$|d(x,A) - d(y,A)| \leqslant \rho_X(x,y)$$

for each $x,y \in X$.

LEMMA 2. ([16]) *The sequence $(d(\cdot,A_n))$ of distance functions is equicontinuous.*

Now we are ready to show the equivalence relation between $\mathscr{I}$-Wijsman convergence and $\mathscr{I}$-α-convergence.

THEOREM 1. *Let $\mathscr{I}$ be an admissible ideal. Then the concepts of $\mathscr{I}$-Wijsman convergence and $\mathscr{I}$-α-convergence are equivalent.*

Proof. First we assume that the sequence (A_n) is $\mathscr{I}$-α-convergent to the set A. Let $\varepsilon > 0$ and $x \in X$. Define $x_n = x$ for each $n \in \mathbb{N}$. Since $A_n \xrightarrow{\mathscr{I}-\alpha} A$, we have

$$\{n \in \mathbb{N} : |d(x_n,A_n) - d(x,A)| \geqslant \varepsilon\} \in \mathscr{I}.$$

Since $d(x_n,A_n) = d(x,A_n)$, we get

$$\{n \in \mathbb{N} : |d(x,A_n) - d(x,A)| \geqslant \varepsilon\} \in \mathscr{I}.$$

Therefore the sequence (A_n) is $\mathscr{I}$-Wijsman convergent to the set A.

On the other hand, now we assume that the sequence (A_n) is $\mathscr{I}$-Wijsman convergent to the set A. Then the sequence $(d(\cdot,A_n))$ of functions is $\mathscr{I}$-convergent to the function $d(\cdot,A)$ on X. Let $x \in X$ and $\varepsilon > 0$. Hence we have

$$K = \left\{n \in \mathbb{N} : |d(x,A_n) - d(x,A)| \geqslant \frac{\varepsilon}{2}\right\} \in \mathscr{I}.$$

By Lemma 2, there exists $\delta = \delta(x,\varepsilon) > 0$ such that

$$|d(y,A_n) - d(x,A_n)| < \frac{\varepsilon}{2} \tag{2}$$

for each $n \in \mathbb{N}$ and each $y \in S(x,\delta)$. Take a sequence (x_n) such that $x_n \longrightarrow x$. In this case, there exists an $n_0 = n_0(x,\delta) \in \mathbb{N}$ such that $\rho_X(x_n,x) < \delta$ for each $n \geqslant n_0$. By the inequality (2), we get

$$|d(x_n,A_n) - d(x,A_n)| < \frac{\varepsilon}{2}$$

for each $n \geqslant n_0$. Therefore we have

$$\begin{aligned} |d(x_n,A_n) - d(x,A)| &\leqslant |d(x_n,A_n) - d(x,A_n)| + |d(x,A_n) - d(x,A)| \\ &< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \end{aligned}$$

for each $n \geqslant n_0$ and $n \in \mathbb{N} \setminus K$. Then

$$\{n \in \mathbb{N} : |d(x_n, A_n) - d(x, A)| \geqslant \varepsilon\} \subseteq K \cup \{1, 2, \ldots, n_0\}$$

and, we get

$$\{n \in \mathbb{N} : |d(x_n, A_n) - d(x, A)| \geqslant \varepsilon\} \in \mathscr{I},$$

since $\mathscr{I}$ is an admissible ideal. Therefore we deduce that the sequence (A_n) of sets is $\mathscr{I}$-α-convergent to the set A, since x is an arbitrary point. □

If we take the ideal $\mathscr{I}_f$ for class $\mathscr{I}$ in Theorem 1, then we have the following corollary.

COROLLARY 2. *The sequence (A_n) Wijsman converges to the set A if and only if for every $x \in X$ and every sequence (x_n) such that $x_n \longrightarrow x$, $\lim d(x_n, A_n) = d(x, A)$.*

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