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BARROW'S INEQUALITY AND SIGNED ANGLE BISECTORS

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Abstract. In this paper we give one extension of BARROW's type inequality in the plane of the triangle $\triangle ABC$ introduce signed angle bisectors.

1. Introduction

Let triangle $\triangle ABC$ be given in Euclidean plane. Denote by R_A, R_B and R_C the distances from the arbitrary point M in the plane of $\triangle ABC$ to the vertices A, B and C respectively, and denote by $\ell_a = |MA'|$, $\ell_b = |MB'|$ and $\ell_c = |MC'|$ the length of angle bisectors of $\angle BMC$, $\angle CMA$ and $\angle AMB$ from the point M respectively (Fig. 1).

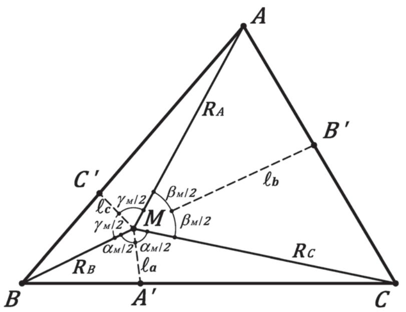

Figure 1: *Barrow's inequality (point M into $\triangle ABC$)*

Barrow's inequality [2]:

$$R_A + R_B + R_C \geqslant 2\,(\ell_a + \ell_b + \ell_c) \tag{1}$$

is true when M is arbitrary point in the interior of triangle $\triangle ABC$. The equality holds iff triangle ABC is equilateral and point M is its circumcenter. In this paper we consider a *Barrow*'s type inequality when M is arbitrary point in the plane of the triangle $\triangle ABC$

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introduce signed angle bisectors. Let us notice that inequalities with angle bisectors recently are considered in papers [1], [6], [7], [15].

Inequality of *Erdös-Mordell* [4]:

$$R_A + R_B + R_C \geqslant 2\left(r_a + r_b + r_c\right) \tag{2}$$

is a consequence of inequality of *Barrow*, where r_a, r_b and r_c are distances of interior point M of triangle to the sides BC, CA and AB respectively.

Let us notice that topic of the *Erdös-Mordell* inequality is current, as it has been shown in recent papers. *V. Pambuccian* proved that, in the plane of absolute geometry, the *Erdös-Mordell* inequality is an equivalent to non-positive curvature [12]. In the paper [11] is given an extension of the *Erdös-Mordell* inequality on the interior of the *Erdös-Mordell* curve. In relation to the *Erdös-Mordell* inequality *N. Dergiades* in the paper [3] proved one extension of the *Erdös-Mordell* type inequality

$$R_A + R_B + R_C \geqslant \left(\frac{c}{b} + \frac{b}{c}\right) r'_a + \left(\frac{c}{a} + \frac{a}{c}\right) r'_b + \left(\frac{a}{b} + \frac{b}{a}\right) r'_c \tag{3}$$

where r'_a, r'_b and r'_c are signed distances of arbitrary point M in the plane triangle to the sides BC, CA and AB respectively.

2. The main results

Proof of *Barrow*'s inequality in the paper of *Z. Lu* [10] is based on the next theorem.

STATEMENT 1. *Let* $p, q, r \geqslant 0$ *and* $\alpha + \beta + \gamma = \pi$. *Then we have the inequality*

$$p + q + r \geqslant 2\sqrt{qr}\cos\alpha \; + 2\sqrt{pr}\cos\beta \; + 2\sqrt{pq}\cos\gamma. \tag{4}$$

Peculiarity of *Barrow*'s and *Lu*'s proofs are, that is, primarily algebraic. In *Lu*'s proof, *Barrow*'s inequality follows from positivity of quadratic function $f(x) = x^2 - 2\left(\sqrt{r}\cos\beta + \sqrt{q}\;\cos\gamma\right)x + q + r - 2\sqrt{qr}\cos\alpha$ in the point $x = \sqrt{p}$ with an appropriate geometric interpretation for p, q, r and α, β, γ (for details see [10]).

In this paper we also give one algebraic proof with geometric interpretation for points outside of the triangle $\triangle ABC$. The following theorems are true.

STATEMENT 2. *Let* $p, q, r \geqslant 0$ *and* $\alpha = \beta + \gamma$. *Then we have the inequality*

$$p + q + r \geqslant -2\sqrt{qr}\cos\alpha \; + 2\sqrt{pr}\cos\beta \; + 2\sqrt{pq}\cos\gamma. \tag{5}$$

Proof. Let us consider the quadratic function

$$g(x) = x^2 - 2\left(\sqrt{r}\cos\beta + \sqrt{q}\cos\gamma\right)x + q + r + 2\sqrt{qr}\cos\alpha. \tag{6}$$

Then a quarter of the discriminant is

$$\frac{1}{4}\delta = \left(\sqrt{r}\cos\beta + \sqrt{q}\cos\gamma\right)^2 - (q+r+2\sqrt{qr}\cos\alpha). \tag{7}$$

Based on $\alpha = \beta+\gamma$ we have $\cos\alpha = \cos(\beta+\gamma) = \cos\beta\cos\gamma - \sin\beta\sin\gamma$ and hence

$$\begin{aligned}\frac{1}{4}\delta &= r\cos^2\beta + q\cos^2\gamma + 2\sqrt{rq}\cos\beta\cos\gamma - q - r - 2\sqrt{rq}\cos\alpha\\ &= r\cos^2\beta + q\cos^2\gamma + 2\sqrt{rq}\cos\beta\cos\gamma - q - r - 2\sqrt{rq}\cos(\beta+\gamma)\\ &= -r\sin^2\beta - q\sin^2\gamma + 2\sqrt{rq}\cos\beta\cos\gamma - 2\sqrt{rq}\cos\beta\cos\gamma + 2\sqrt{rq}\sin\beta\sin\gamma.\end{aligned}$$

Using previous identity we obtained

$$\delta = -4\left(\sqrt{r}\sin\beta - \sqrt{q}\sin\gamma\right)^2 < 0,$$

hence $g(x) \geqslant 0$. Finally, letting $x = \sqrt{p}$ we obtained (5). □

REMARK 1. Let us emphasize that for term $A = p+q+r+2\sqrt{qr}\cos\alpha - 2\sqrt{pr}\cos\beta - 2\sqrt{pq}\cos\gamma$, when $\gamma = \alpha-\beta$, follows inequality

$$A = \left(\sqrt{r} - \sqrt{p}\cos\beta + \sqrt{q}\cos\alpha\right)^2 + (\sqrt{p}\sin\beta - \sqrt{q}\sin\alpha)^2 \geqslant 0,$$

analogously using the *Lagrange*'s complete square identity from [8], [9]. Therefore we have second proof of inequality (5).

STATEMENT 3. *Let* $p,q,r \geqslant 0$ *and* $\alpha = \beta+\gamma$. *Then we have the inequality*

$$p+q+r \geqslant 2\sqrt{qr}\cos\alpha \; - 2\sqrt{pr}\cos\beta \; - 2\sqrt{pq}\cos\gamma. \tag{8}$$

Proof. Let us consider the term $A = p+q+r-2\sqrt{qr}\cos\alpha + 2\sqrt{pr}\cos\beta + 2\sqrt{pq}\cos\gamma$, for $\gamma = \alpha-\beta$. Notice that for the term A, by the *Lagrange*'s complete square identity, the following two representations are true.

1°. If $\frac{\pi}{2} \leqslant \alpha < \pi$, then $\cos\alpha \leqslant 0$, and therefore

$$A = \left(\sqrt{r} + \sqrt{p}\cos\beta + \sqrt{q}\cos\alpha\right)^2 + (\sqrt{p}\sin\beta + \sqrt{q}\sin\alpha)^2 - 4\sqrt{qr}\cos\alpha \geqslant 0. \tag{9}$$

2°. If $0 < \alpha < \frac{\pi}{2}$, then $\cos\alpha > 0$. From $\alpha = \beta+\gamma$ follows $\cos\beta > 0$, and therefore

$$A = \left(\sqrt{r} - \sqrt{p}\cos\beta - \sqrt{q}\cos\alpha\right)^2 + (\sqrt{p}\sin\beta + \sqrt{q}\sin\alpha)^2 + 4\sqrt{pr}\cos\beta \geqslant 0. \tag{10}$$

□

Let us introduce the division of the plane of triangle $\triangle ABC$ to following areas $\lambda_0 = (+,+,+)$, $\lambda_1 = (-,+,+)$, $\lambda_2 = (+,-,+)$, $\lambda_3 = (+,+,-)$, $\lambda_4 = (+,-,-)$,

$\lambda_5 = (-,+,-)$, $\lambda_6 = (-,-,+)$, (Fig. 2), via signes of homogenous barycentric coordinates of a point as given in the paper [14] $\big($see also the Section 7.2 in [5]$\big)$. Then λ_0 is the interior area of the triangle $\triangle ABC$. Let us notice that $(\lambda_0 \cup \lambda_1) \cup (BC)$ is the interior area of the angle $\angle A$, and λ_4 is the interior area of the opposite angle. Analogously $(\lambda_0 \cup \lambda_2) \cup (AC)$ is the interior area of the angle $\angle B$, λ_5 is the interior area of the opposite angle and $(\lambda_0 \cup \lambda_3) \cup (AB)$ is the interior area of the angle $\angle C$, λ_6 is the interior area of the opposite angle.

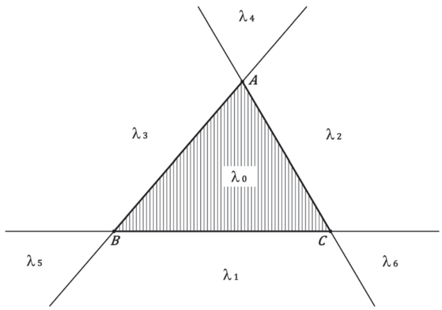


Figure 2: *The division of the plane by the sidelines of the triangle* $\triangle ABC$

The following auxiliary statement is true.

LEMMA 0. *Let B and C be fixed points in the plane and let M be arbitrary point in the plane. For ℓ length of angle bisector of $\angle BMC$ from point M following formulas are true*:

$$\ell = \frac{2R_BR_C}{R_B+R_C}\cos\frac{\alpha_M}{2} = \frac{\sqrt{R_BR_C}}{R_B+R_C}\sqrt{(R_B+R_C)^2-|BC|^2}, \tag{11}$$

where $R_B = |MB|$, $R_C = |MC|$ and $\alpha_M = \angle BMC$. Especially, for $\mathfrak{p}$ line throughout points B and C is true:

$$\ell = \begin{cases} 0 & : \ M \in [BC], \\ \dfrac{2R_BR_C}{R_B+R_C} & : \ M \in \mathfrak{p}\backslash[BC]. \end{cases} \tag{12}$$

In further considerations let $p = R_A$, $q = R_B$, $r = R_C$. Then, *Z. Lu*, in the paper [10], proved the following *Barrow*'s type inequality.

THEOREM 0. [10] *In the area λ_0 the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell_c. \tag{13}$$

REMARK 2. *Barrow*'s inequality is a consequence of the previous inequality.

From previous Lemma follows next auxiliary statement.

LEMMA 1. *(i) If $M = A$, i.e. $R_A = 0$ then*:

$$R_B + R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a. \tag{14}$$

(ii) If $M = B$, i.e. $R_B = 0$ then:

$$R_A + R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b. \tag{15}$$

(iii) If $M = C$, i.e. $R_C = 0$ then:

$$R_A + R_B \geqslant \left(\frac{\sqrt{R_B}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_B}}\right)\ell_c. \tag{16}$$

Denote with $\mathtt{cl}$ closure of a plane set. The following theorem is true.

THEOREM 1. *In the area $\mathtt{cl}\,(\lambda_1)\setminus\{B,C\}$ the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}}\right)(-\ell_a) + \left(\frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b + \left(\frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell_c. \tag{17}$$

Proof. Let $M \in \mathtt{cl}\,(\lambda_1)\setminus\{B,C\}$, then $\alpha_M = \beta_M + \gamma_M$ i.e. $\frac{\alpha_M}{2} = \frac{\beta_M}{2} + \frac{\gamma_M}{2}$ (Fig. 3).

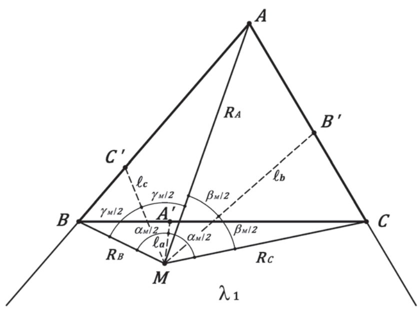


Figure 3: *Extension of the Barrow's inequality for the point $M \in \mathtt{cl}\,(\lambda_1)\setminus\{B,C\}$*

Based on the Statement 2, the following inequality holds

$$R_A+R_B+R_C \geqslant -2\sqrt{R_BR_C}\cos\frac{\alpha_M}{2} + 2\sqrt{R_AR_C}\cos\frac{\beta_M}{2} + 2\sqrt{R_AR_B}\cos\frac{\gamma_M}{2}. \tag{18}$$

Based on Lemma 0 from previous inequality we obtained

$$\begin{aligned} R_A+R_B+R_C &\geqslant -\frac{R_B+R_C}{\sqrt{R_BR_C}}\ \ell_a + \frac{R_A+R_C}{\sqrt{R_AR_C}}\ \ell_b + \frac{R_A+R_B}{\sqrt{R_AR_B}}\ \ell_c \\ &= \left(\frac{\sqrt{R_C}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_C}}\right)(-\ell_a) + \left(\frac{\sqrt{R_C}}{\sqrt{R_A}} + \frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b + \left(\frac{\sqrt{R_A}}{\sqrt{R_B}} + \frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell_c. \end{aligned} \tag{19}$$

□

Next two theorems are direct consequence of the Statement 2 by following cyclic replacements $\alpha_M \mapsto \beta_M$, $\beta_M \mapsto \gamma_M$, $\gamma_M \mapsto \alpha_M$ and $R_A \mapsto R_B$, $R_B \mapsto R_C$, $R_C \mapsto R_A$ respectively.

THEOREM 2. *In the area* $\mathtt{cl}\,(\lambda_2)\backslash\{A,C\}$ *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)(-\ell_b)+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell_c. \quad (20)$$

THEOREM 3. *In the area* $\mathtt{cl}\,(\lambda_3)\backslash\{A,B\}$ *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)(-\ell_c). \quad (21)$$

The following theorem is true.

THEOREM 4. *In the area* λ_4 *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)(-\ell_b)+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)(-\ell_c). \quad (22)$$

Proof. Let $M\in\lambda_4$, then $\alpha_M=\beta_M+\gamma_M$ i.e. $\frac{\alpha_M}{2}=\frac{\beta_M}{2}+\frac{\gamma_M}{2}$. Based on the Statement 3 the following inequality is true

$$R_A+R_B+R_C \geqslant 2\sqrt{R_BR_C}\cos\frac{\alpha_M}{2}-2\sqrt{R_AR_C}\cos\frac{\beta_M}{2}-2\sqrt{R_AR_B}\cos\frac{\gamma_M}{2}. \quad (23)$$

Substitutions

$$\ell_a=|\,MA'\,|=2\frac{R_BR_C}{R_B+R_C}\cos\frac{\alpha_M}{2}, \quad (24)$$

$$\ell_b=|\,MB'\,|=2\frac{R_AR_C}{R_A+R_C}\cos\frac{\beta_M}{2}, \quad (25)$$

$$\ell_c=|\,MC'\,|=2\frac{R_AR_B}{R_A+R_B}\cos\frac{\gamma_M}{2} \quad (26)$$

in (23) give

$$\begin{aligned}R_A+R_B+R_C &\geqslant \frac{R_B+R_C}{\sqrt{R_BR_C}}\;\ell_a-\frac{R_A+R_C}{\sqrt{R_AR_C}}\;\ell_b-\frac{R_A+R_B}{\sqrt{R_AR_B}}\;\ell_c\\ &=\left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)(-\ell_b)+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)(-\ell_c).\end{aligned} \quad (27)$$

□

Next two theorems are direct consequence of the Statement 3 by following cyclic replacements $\alpha_M\mapsto\beta_M$, $\beta_M\mapsto\gamma_M$, $\gamma_M\mapsto\alpha_M$ and $R_A\mapsto R_B$, $R_B\mapsto R_C$, $R_C\mapsto R_A$ respectively.

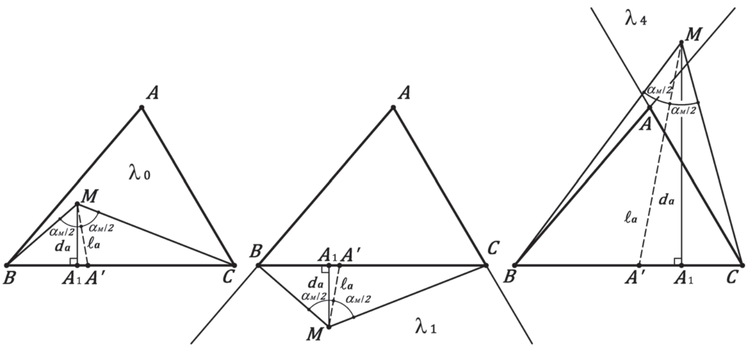
THEOREM 5. *In the area* λ_5 *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)(-\ell_a)+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell_b+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)(-\ell_c). \quad (28)$$

THEOREM 6. *In the area* λ_6 *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)(-\ell_a)+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)(-\ell_b)+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell_c. \quad (29)$$

Now, we give definition of the signed angle bisector for the point M in the plane of the triangle $\triangle ABC$. Let be A fixed vertex and let $\mathfrak{p}$ be line through vertices B and C. Denote $d=|MA_1|$ distance of the point M to the line $\mathfrak{p}$ and let $\ell=|MA'|$ be length of the bisector of the angle $\angle BMC$. If d' be signed distance of the point M to the line $\mathfrak{p}$ related to the vertex A [13] (p. 308.), then $d'=+d$ if M and A with same side of line $\mathfrak{p}$, otherwise $d'=-d$. Let us define signed angle bisector ℓ' analogously $\ell'=+\ell$ if M and A with same side of line $\mathfrak{p}$, otherwise $\ell'=-\ell$ (Fig. 4). In the case $M\in\mathfrak{p}$ then $d'=0$ and then ℓ' given by formula (12).



$d'_a=+d_a$ $\qquad$ $d'_a=-d_a$ $\qquad$ $d'_a=+d_a$

$\ell'_a=+\ell_a$ $\qquad$ $\ell'_a=-\ell_a$ $\qquad$ $\ell'_a=+\ell_a$

Figure 4: *Signed distances and signed angle bisectors*

Let us denote $\mu_1=\mathrm{cl}(\lambda_1)\setminus\{B,C\}$, $\mu_2=\mathrm{cl}(\lambda_2)\setminus\{A,C\}$, $\mu_3=\mathrm{cl}(\lambda_3)\setminus\{A,B\}$, $\mu_4=\lambda_4$, $\mu_5=\lambda_5$ and $\mu_6=\lambda_6$. Then $\bigcup_{i=1}^{6}\mu_i\cup\{A,B,C\}$ is a complete division of the plane of the triangle $\triangle ABC$. Finally, analogously to *Dergiades* extension of the *Erdös-Mordell* inequality [3], from previous theorems, an extension of *Barrow*'s type inequality (13) is obtained by the following theorem.

STATEMENT 4. *For the point* $M\in\bigcup\limits_{i=1}^{6}\mu_i$ *the following inequality is true*:

$$R_A+R_B+R_C \geqslant \left(\frac{\sqrt{R_C}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_C}}\right)\ell'_a+\left(\frac{\sqrt{R_C}}{\sqrt{R_A}}+\frac{\sqrt{R_A}}{\sqrt{R_C}}\right)\ell'_b+\left(\frac{\sqrt{R_A}}{\sqrt{R_B}}+\frac{\sqrt{R_B}}{\sqrt{R_A}}\right)\ell'_c; \quad (30)$$

otherwise for points $M=A$, $M=B$, $M=C$ *following inequalities* (14), (15), (16) *are true respectively.*

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