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UNIVALENCY OF A NONLINEAR INTEGRAL OPERATOR OF ANALYTIC FUNCTIONS

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Abstract. In this paper, we obtain new univalence conditions for the nonlinear integral operator $F_\alpha(p)(z) = \left[\alpha \int_0^z u^{\alpha-1} \exp\left(\int_0^u \frac{\beta(p(t)-1)}{t} dt\right) du\right]^{\frac{1}{\alpha}}$ where $p(z)$ is analytic function in the open unit disk and satisfies $p(0) = 1$, $\alpha \in \mathbb{C}$ with $\mathrm{Re}(\alpha) > 0$ and $\beta \in \mathbb{C}^* = \mathbb{C}\backslash\{0\}$. The numbers of known or new univalence conditions are shown to follow upon specializing the parameters involved in our main results.

1. Introduction and definitions

Let $\mathscr{A}$ denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic in the open unit disk $\mathscr{U} = \{z \in \mathbb{C} : |z| < 1\}$, and let $\mathscr{S}$ denote the subclass of $\mathscr{A}$ consisting of all functions $f(z) \in \mathscr{A}$ which are univalent in $\mathscr{U}$. Furthermore, let $\mathscr{P}$ be the class of functions $p(z)$ of which are analytic in $\mathscr{U}$ and satisfy $p(0) = 1$.

Very recently, Attiya [1] introduced the nonlinear operator defined by

$$F_\alpha(q)(z) := \left[\alpha \int_0^z u^{\alpha-1} \exp\left(\int_0^u \frac{q(t)}{t} dt\right) du\right]^{\frac{1}{\alpha}} \qquad (z \in \mathscr{U}), \tag{1.2}$$

where the function $q \in \mathscr{A}$ with $q(0) = 0$ and $\alpha \in \mathbb{C}$ with $\mathrm{Re}(\alpha) > 0$.

Let $F_{\alpha,\beta}(p) : \mathscr{P} \to \mathscr{A}$ be the nonlinear integral operator defined by

$$F_{\alpha,\beta}(p)(z) = \left[\alpha \int_0^z u^{\alpha-1} \exp\left(\int_0^u \frac{\beta(p(t)-1)}{t} dt\right) du\right]^{\frac{1}{\alpha}}. \tag{1.3}$$

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where $p \in \mathscr{P}$, $\alpha \in \mathbb{C}$ with $\operatorname{Re}(\alpha) > 0$ and $\beta \in \mathbb{C}^* = \mathbb{C} \backslash \{0\}$. The exponential function in the above relation is the principal value.

The problem of finding sufficient conditions for univalence of various integral operators has been investigated in many recent works (see, for example, [2]–[10], [12], [14]).

In our present investigation, we study the univalence conditions for the integral operator $F_\alpha(p)$ defined by (1.3). Here and throughout in the sequel every many-valued function is taken with the principal branch.

In order to derive our main results, we have to recall here the following lemmas.

LEMMA 1.1. ([11]) *If $p(z) \in \mathscr{P}$, then*

$$\frac{1-|z|}{1+|z|} \leqslant |p(z)| \leqslant \frac{1+|z|}{1-|z|} \qquad (z \in \mathscr{U}). \tag{1.4}$$

LEMMA 1.2. ([17]) *Let $\lambda \geqslant 0$ and $M > 0$. If $p(z) \in \mathscr{P}$ satisfies*

$$\left|p(z) + \lambda z p'(z) - 1\right| < M \qquad (z \in \mathscr{U}),$$

then we have

$$|p(z) - 1| < \frac{M}{1+\lambda} \qquad (z \in \mathscr{U}). \tag{1.5}$$

The result in (1.5) cannot be improved.

Also, in the proofs of our main results we need the following univalence criterion.

LEMMA 1.3. ([13]) *Let α be complex number with $\operatorname{Re}(\alpha) > 0$. If $h \in \mathscr{A}$ satisfies*

$$\frac{1-|z|^{2\operatorname{Re}(\alpha)}}{\operatorname{Re}(\alpha)} \left|\frac{zh''(z)}{h'(z)}\right| \leqslant 1,$$

for all $z \in \mathscr{U}$ then the integral operator

$$F_\alpha(z) = \left\{ \alpha \int_0^z t^{\alpha-1} h'(t)\,dt \right\}^{\frac{1}{\alpha}} \tag{1.6}$$

is in the class $\mathscr{S}$.

2. Univalence condition for $F_{\alpha,\beta}(p)$

We begin with the following sufficient condition for integral operator $F_{\alpha,\beta}(p)$ to be analytic and univalent in $\mathscr{U}$.

THEOREM 2.1. *Let the function $p \in \mathscr{P}$. If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ with*

$$|\beta| \leqslant \begin{cases} \frac{\mathrm{Re}(\alpha)}{4}, & \text{if } 0 < \mathrm{Re}(\alpha) < 1 \\ \frac{1}{4}, & \text{if } \mathrm{Re}(\alpha) \geqslant 1. \end{cases} \tag{2.1}$$

then the integral operator $F_{\alpha,\beta}(p)(z)$ defined by (1.3) is in the class $\mathscr{S}$.

Proof. Define the function $h(z)$ by

$$h(z) = \int_0^z \exp\left(\int_0^u \frac{\beta(p(t)-1)}{t} dt\right) du. \tag{2.2}$$

Then it is easy to see that $h(0) = h'(0) - 1 = 0$ and

$$\frac{zh''(z)}{h'(z)} = \beta(p(z)-1). \tag{2.3}$$

Hence from (2.3) and (1.4), we get

$$\begin{aligned} \frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)} \left|\frac{zh''(z)}{h'(z)}\right| &= \frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)} |\beta| \, |p(z)-1| \\ &\leqslant \frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)} |\beta| \left(\frac{1+|z|}{1-|z|}+1\right) \\ &\leqslant \frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)} |\beta| \frac{2}{1-|z|}, \end{aligned} \tag{2.4}$$

for all $z \in \mathscr{U}$.

Let $0 < \mathrm{Re}(\alpha) < 1$. Then, we observe that the function $\phi : (0,1) \to \mathbb{R}$, $\phi(\zeta) = 1 - b^{2\zeta}$; $(0 < b < 1)$ is increasing function. Thus, for $b = |z|$; $z \in \mathscr{U}$, we obtain

$$1 - |z|^{2\mathrm{Re}(\alpha)} \leqslant 1 - |z|^2 \tag{2.5}$$

for all $z \in \mathscr{U}$. If $\mathrm{Re}(\alpha) \geqslant 1$, the function $\psi : [1,\infty) \to \mathbb{R}$, $\psi(\zeta) = \frac{1-b^{2\zeta}}{\zeta}$; $(0 < b < 1)$ is decreasing function, then for $b = |z|$; $z \in \mathscr{U}$, we have

$$\frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)} \leqslant 1 - |z|^2. \tag{2.6}$$

for all $z \in \mathscr{U}$. From (2.4)–(2.6), and the hypothesis (2.1), we get

$$\frac{1-|z|^{2\mathrm{Re}(\alpha)}}{\mathrm{Re}(\alpha)}\left|\frac{zh''(z)}{h'(z)}\right| \leqslant \begin{cases} \frac{4|\beta|}{\mathrm{Re}(\alpha)}, & \text{if } 0<\mathrm{Re}(\alpha)<1 \\ 4|\beta|, & \text{if } \mathrm{Re}(\alpha)\geqslant 1, \end{cases}$$
$$\leqslant 1$$

for all $z \in \mathscr{U}$. Applying Lemma 1.3 for the function $h(z)$, we prove that $F_{\alpha,\beta}(p)(z) \in \mathscr{S}$. This evidently completes the proof of Theorem 2.1. □

THEOREM 2.2. *Let the function $p \in \mathscr{P}$ satisfies*

$$\left|p(z)+\lambda zp'(z)-1\right| < M \qquad (\lambda \geqslant 0,\ M>0;\ z\in\mathscr{U}). \tag{2.7}$$

If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ with

$$|\beta| \leqslant \frac{1}{2M}(1+\lambda)(2a+1)^{\frac{2a+1}{2a}} \qquad (\mathrm{Re}(\alpha)=a>0), \tag{2.8}$$

then the integral operator $F_{\alpha,\beta}(p)(z)$ defined by (1.3) is in the class $\mathscr{S}$.

Proof. In view of Schwarz' Lemma and (1.5), we have

$$|p(z)-1| \leqslant \frac{M}{1+\lambda}|z| \quad (z\in\mathscr{U}). \tag{2.9}$$

Hence from (2.3) and (2.9), we get

$$\begin{aligned}\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| &= \frac{1-|z|^{2a}}{a}|\beta|\,|p(z)-1| \\ &\leqslant \left(\frac{M}{1+\lambda}\right)|\beta|\frac{|z|\left(1-|z|^{2a}\right)}{a}, \qquad (z\in\mathscr{U}).\end{aligned} \tag{2.10}$$

Let us denote $|z|=x,\ x\in[0,1],\ \mathrm{Re}(\alpha)=a>0$ and $\Psi(x)=x(1-x^{2a})$. It is easy to prove that the maximum is attained at the point $x=1/(2a+1)^{1/2a}$ and thus we have

$$\Psi(x) \leqslant \frac{2a}{(2a+1)^{\frac{2a+1}{2a}}}.$$

In view of this inequality and (2.10), we obtain

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leqslant \frac{2M|\beta|}{(1+\lambda)(2a+1)^{\frac{2a+1}{2a}}} \qquad (z\in\mathscr{U}).$$

It follows from (2.8) that

$$\frac{1-|z|^{2a}}{a}\left|\frac{zh''(z)}{h'(z)}\right| \leqslant 1 \qquad (z\in\mathscr{U}).$$

Applying Lemma 1.3 for the function $h(z)$, we prove that $F_{\alpha,\beta}(p)(z) \in \mathscr{S}$. The proof is completed. □

3. Some consequences of the main results

In this section, we give some applications of the results produced in the second section.

Letting $p(z) = zf'(z)/f(z)$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.1. *Let the function* $f \in \mathscr{A}$. *If* $\alpha \in \mathbb{C}$ *and* $\beta \in \mathbb{C}^*$ *with*

$$|\beta| \leqslant \begin{cases} \frac{\mathrm{Re}(\alpha)}{4}, & \text{if } 0 < \mathrm{Re}(\alpha) < 1 \\ \frac{1}{4}, & \text{if } \mathrm{Re}(\alpha) \geqslant 1, \end{cases} \tag{3.1}$$

then the integral operator

$$H_{\alpha,\beta}(f)(z) = \left[\alpha \int_0^z u^{\alpha-1} \left(\frac{f(u)}{u}\right)^{\beta} du\right]^{\frac{1}{\alpha}} \tag{3.2}$$

is in the class $\mathscr{S}$.

COROLLARY 3.2. *Let the function* $f \in \mathscr{A}$ *satisfy*

$$\left|zf'(z)/f(z) + \lambda z\left(zf'(z)/f(z)\right)' - 1\right| < M \qquad (\lambda \geqslant 0,\ M > 0;\ z \in \mathscr{U}). \tag{3.3}$$

If $\alpha \in \mathbb{C}$ *and* $\beta \in \mathbb{C}^*$ *with*

$$|\beta| \leqslant \frac{1}{2M}(1+\lambda)(2a+1)^{\frac{2a+1}{2a}} \qquad (\mathrm{Re}(\alpha) = a > 0), \tag{3.4}$$

then the integral operator $H_{\alpha,\beta}(f)(z)$ *defined by (3.2) is in the class* $\mathscr{S}$.

If we take $\alpha = 1$ and $\lambda = 0$ in Corollary 3.2, we immediately obtain the following result due to Pescar [15].

COROLLARY 3.3. *Let the function* $f \in \mathscr{A}$ *satisfies*

$$\left|\frac{zf'(z)}{f(z)} - 1\right| < \frac{3\sqrt{3}}{2|\beta|} \qquad (\beta \in \mathbb{C}^*;\ z \in \mathscr{U}), \tag{3.5}$$

then $\int_0^z (f(u)/u)^{\beta} du \in \mathscr{S}$.

Letting $p(z) = 1 + zf''(z)/f'(z)$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.4. *Let the function $f \in \mathscr{A}$. If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ satisfy the inequality (3.1), then the integral operator*

$$Q_{\alpha,\beta}(f)(z) = \left[\alpha \int_0^z u^{\alpha-1}\left(f'(u)\right)^{\beta}\, du\right]^{\frac{1}{\alpha}} \tag{3.6}$$

is in the class $\mathscr{S}$.

COROLLARY 3.5. *Let the function $f \in \mathscr{A}$ satisfies*

$$\left|zf''(z)/f'(z) + \lambda z\left(1 + zf''(z)/f'(z)\right)'\right| < M \qquad (\lambda \geqslant 0,\ M > 0;\ z \in \mathscr{U}). \tag{3.7}$$

If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ satisfy the inequality (3.4), then the integral operator $Q_{\alpha,\beta}(f)(z)$ defined by (3.6) is in the class $\mathscr{S}$.

By setting $\alpha = 1$ and $\lambda = 0$ in Corollary 3.5, we immediately obtain the following result due to Pescar [15].

COROLLARY 3.6. *Let the function $f \in \mathscr{A}$ satisfies*

$$\left|\frac{zf''(z)}{f'(z)}\right| < \frac{3\sqrt{3}}{2|\beta|} \qquad (\beta \in \mathbb{C}^*;\ z \in \mathscr{U}), \tag{3.8}$$

then $\int_0^z (f'(u))^{\beta}\, du \in \mathscr{S}$.

Letting $p(z) = zf'(z) + 1$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.7. *Let the function $f \in \mathscr{A}$. If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ satisfy the inequality (3.1), then the integral operator*

$$K_{\alpha,\beta}(f)(z) = \left[\alpha \int_0^z u^{\alpha-1}\left(e^{f(u)}\right)^{\beta}\, du\right]^{\frac{1}{\alpha}} \tag{3.9}$$

is in the class $\mathscr{S}$.

COROLLARY 3.8. *Let the function $f \in \mathscr{A}$ satisfies*

$$\left|zf'(z) + \lambda\left(z^2 f''(z) + zf'\right)\right| < M \qquad (\lambda \geqslant 0,\ M > 0;\ z \in \mathscr{U}). \tag{3.10}$$

If $\alpha \in \mathbb{C}$ and $\beta \in \mathbb{C}^*$ satisfy the inequality (3.1), then the integral operator $K_{\alpha,\beta}(f)(z)$ defined by (3.9) is in the class $\mathscr{S}$.

By setting $\lambda = 0$ in Corollary 3.8, we immediately obtain the following result due to Attiya [1].

COROLLARY 3.9. *Let the function* $f \in \mathscr{A}$ *satisfies*

$$\left|zf'(z)\right| < M \qquad (M > 0;\ z \in \mathscr{U}). \tag{3.11}$$

If $\alpha \in \mathbb{C}$ *and* $\mathrm{Re}(\beta) > 0$ *with*

$$|\beta| \leqslant \frac{1}{2M}(2a+1)^{\frac{2a+1}{2a}} \qquad (\mathrm{Re}(\alpha) = a > 0), \tag{3.12}$$

then the integral operator $K_{\alpha,\beta}(f)(z)$ *defined by (3.9) is in the class* $\mathscr{S}$.

By setting $\lambda = 0$ in Corollary 3.9, we immediately obtain the following result due to Pescar [16].

COROLLARY 3.10. *Let the function* $f \in \mathscr{A}$ *satisfies*

$$\left|zf'(z)\right| < 1 \qquad (z \in \mathscr{U}) \tag{3.13}$$

If $\alpha \in \mathbb{C}$; $\mathrm{Re}(\alpha) = a > 0$ *with*

$$|\alpha| \leqslant \frac{1}{2}(2a+1)^{\frac{2a+1}{2a}} \tag{3.14}$$

then the integral operator

$$T_\alpha(f)(z) = \left[\alpha \int_0^z u^{\alpha-1}\left(e^{f(u)}\right)^{\alpha} du\right]^{\frac{1}{\alpha}} \tag{3.15}$$

is in the class $\mathscr{S}$.

If we take $\alpha = 1$, in Corollary 3.9, then we obtain the following result.

COROLLARY 3.11. *Let the function* $f \in \mathscr{A}$ *satisfies*

$$\left|zf'(z)\right| < \frac{3\sqrt{3}}{2|\beta|} \qquad (\beta \in \mathbb{C}^*;\ z \in \mathscr{U}), \tag{3.16}$$

then $\int_0^z \left(e^{f(u)}\right)^{\beta} du \in \mathscr{S}$.

Letting $p(z) = \sum_{j=1}^{n} \gamma_j \left(\frac{zf_j'(z)}{f_j(z)} - 1\right) + 1$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.12. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1, \ldots, n)$ *and* $\alpha \in \mathbb{C}$. *If* $\beta \in \mathbb{C}^*$ *satisfies the inequality (3.1), then the integral operator*

$$D_{\alpha,\beta_1,\cdots,\beta_n}(f)(z) = \left[\alpha \int_0^z u^{\alpha-1} \prod_{j=1}^{n}\left(\frac{f_j(u)}{u}\right)^{\beta\gamma_j} du\right]^{\frac{1}{\alpha}} \tag{3.17}$$

is in the class $\mathscr{S}$.

COROLLARY 3.13. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1,\dots,n)$ *satisfy*

$$\left|\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j'(z)}{f_j(z)}-1\right)+\lambda z\left(\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j'(z)}{f_j(z)}-1\right)\right)'\right|<M \qquad (\lambda \geqslant 0,\ M>0;\ z\in\mathscr{U}). \tag{3.18}$$

Also, let $\alpha \in \mathbb{C}$ *with* $\operatorname{Re}(\alpha) = a > 0$ *and* $\beta \in \mathbb{C}^*$ *satisfy the inequality (3.4), then the integral operator* $D_{\alpha,\beta_1,\cdots,\beta_n}(f)(z)$ *defined by (3.17) is in the class* $\mathscr{S}$.

Letting $p(z) = \sum_{j=1}^{n}\gamma_j\left(\frac{zf_j''(z)}{f_j'(z)}\right)+1$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.14. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1,\dots,n)$ *and* $\alpha \in \mathbb{C}$. *If* $\beta \in \mathbb{C}^*$ *satisfies the inequality (3.1), then the integral operator*

$$I_{\alpha,\beta_1,\cdots,\beta_n}(f)(z) = \left[\alpha\int_0^z u^{\alpha-1}\prod_{j=1}^{n}(f_j'(u))^{\beta\gamma_j}\,du\right]^{\frac{1}{\alpha}} \tag{3.19}$$

is in the class $\mathscr{S}$.

COROLLARY 3.15. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1,\dots,n)$ *satisfy*

$$\left|\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j''(z)}{f_j'(z)}\right)+\lambda z\left(\sum_{j=1}^{n}\gamma_j\left(\frac{zf_j''(z)}{f_j'(z)}\right)\right)'\right|<M \qquad (\lambda \geqslant 0,\ M>0;\ z\in\mathscr{U}). \tag{3.20}$$

Also, let $\alpha \in \mathbb{C}$ *with* $\operatorname{Re}(\alpha) = a > 0$ *and* $\beta \in \mathbb{C}^*$ *satisfy the inequality (3.4), then the integral operator* $I_{\alpha,\beta_1,\cdots,\beta_n}(f)(z)$ *defined by (3.19) is in the class* $\mathscr{S}$.

Finally, by setting $p(z) = \sum_{j=1}^{n}\gamma_j(zf_j'(z))+1$; $(j = 1,\dots,n)$ in Theorems 2.1 and 2.2, we obtain the next two corollaries.

COROLLARY 3.16. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1,\dots,n)$ *and* $\alpha \in \mathbb{C}$. *If* $\beta \in \mathbb{C}^*$ *satisfies the inequality (3.1), then the integral operator*

$$\mathscr{G}_{\alpha,\beta_1,\cdots,\beta_n}(f)(z) = \left[\alpha\int_0^z u^{\alpha-1}\prod_{j=1}^{n}\left(e^{f_j(u)}\right)^{\beta\gamma_j}\,du\right]^{\frac{1}{\alpha}} \tag{3.21}$$

is in the class $\mathscr{S}$.

COROLLARY 3.17. *Let the functions* $f_j \in \mathscr{A}$, $\gamma_j \in \mathbb{C}^*$; $(j = 1,\dots,n)$ *satisfy*

$$\left|\sum_{j=1}^{n}\gamma_j(zf_j'(z))+\lambda z\left(\sum_{j=1}^{n}\gamma_j(zf_j'(z))\right)'\right|<M \qquad (\lambda \geqslant 0,\ M>0;\ z\in\mathscr{U}). \tag{3.22}$$

Also, let $\alpha \in \mathbb{C}$ *with* $\mathrm{Re}(\alpha) = a > 0$ *and* $\beta \in \mathbb{C}^*$ *satisfy the inequality (3.4), then the integral operator* $\mathcal{G}_{\alpha,\beta_1,\cdots,\beta_n}(f)(z)$ *defined by (3.21) is in the class* $\mathcal{S}$.

REMARK 3.18. If we put $n = 1$, $\gamma_1 = 1$ and $f_1 = f$ in Corollaries 3.12–3.17, we obtain Corollaries 3.1, 3.2, 3.4, 3.5, 3.7 and 3.8, respectively.

REFERENCES

[1] A. A. ATTIYA, *Nonlinear operator and the sufficient conditions of univalence with applications*, Journal of Mathematical Inequalities (In press).

[2] Á. BARICZ AND B. A. FRASIN, *Univalence of integral operators involving Bessel functions*, Applied Mathematics Letters, Volume 23, Issue 4, 2010, Pages 371–376.

[3] D. BLEZU AND R. N. PASCU, *Univalence criteria for integral operators*, Glasnik Math. **36** (56) (2001), 241–245.

[4] D. BREAZ AND N. BREAZ, *Univalence conditions for certain integral operators*, Studia Universitatis Babes-Bolyai, Mathematica, Vol. XLVII No. 2 (2002), 9–15.

[5] D. BREAZ AND V. PESCAR, *Univalence conditions for some general integral operators*, Banach J. Math. Anal. (1) **2** (2008), 53–58.

[6] S. BULUT, *Univalence preserving integral operators defined by generalized Al-Oboudi differential operators*, An. Şt. Univ. Ovidius Constanţa, **17** (1), 2009, 37–50.

[7] B. A. FRASIN, *Univalence of two general integral operator*, Filomat, Vol. 23, Number 3, (2009), 223–229.

[8] B. A. FRASIN, *Order of convexity and univalency of general integral operator*, Journal of the Franklin Institute, Volume 348, Issue 6, 2011, 1013–1019.

[9] B. A. FRASIN, *New general integral operator*, Computers and Mathematics with Applications, **62** (2011) 4272–4276.

[10] B. A. FRASIN, M. DARUS AND M. ARIF, *Convexity and univalency of general integral operator*, Proc. Jangjeon Math. Soc. 16 (2013), No. 4. pp. 465–437.

[11] A. W. GOODMAN, *Univalent Functions*, Vol. 1, Mariner, Tampa, Florida (1983).

[12] G. I. OROS, G. OROS AND D. BREAZ, *Sufficient conditions for univalence of an integral operator*, J. Ineq. Appl., Vol. 2008, Article ID 127645, 7 pages.

[13] N. N. PASCU, *On a univalence criterion, II*, in: Itinerant Seminar on Functional Equations, Approximation and Convexity (Cluj-Napoca, 1985), pp. 153–154, Preprint 86–6, Univ. Babes-Bolyai, Cluj-Napoca, 1985.

[14] V. PESCAR, *Univalence conditions for certain integral operators*, Journal of Inequalities in Pure and Applied Mathematics, Volume 7, Issue 4, Article 147.

[15] V. PESCAR, *The univalence and the convexity properties for a new integral operator*, Stud. Univ. Babeş-Bolyai Math. **56** (2011), No. 4, 65–69.

[16] V. PESCAR, *Simple sufficient conditions for univalence*, Stud. Univ. Babeş-Bolyai Math.Volume XLIX, Number 2, June 2004.

[17] Z. ZHONGZHU AND S. OWA, *Convolution properties of a class of bounded analytic functions*, Bulletin of The Australian Mathematical Society, Volume 45, (1992), 9–23.

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