Journal of
Mathematical
Inequalities
Volume 9, Number 3 (2015), 951–959
doi:10.7153/jmi-09-78

THE LOGARITHMIC COEFFICIENT INEQUALITY FOR CLOSE–TO–CONVEX FUNCTIONS OF COMPLEX ORDER

MURAT ÇAĞLAR

(*Communicated by J. Pečarić*)

Abstract. We prove that if $n \geqslant 2$ for each close-to-convex functions of complex order b in $\mathscr{S}$ whose $n-th$ logarithmic coefficients γ_n satisfies $|\gamma_n| \leqslant An^{-1}\log n$, where A is an absolute constant.

1. Introduction

Let $\mathscr{A}$ denote the class of functions f analytic in the unit disk $\mathscr{U} = \{z \in \mathbb{C} : |z| < 1\}$ having the power series

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad z \in \mathscr{U}. \tag{1.1}$$

Let $\mathscr{S}$ denote the class of functions $f \in \mathscr{A}$ which are univalent in $\mathscr{U}$ and $\mathscr{S}^*$ point out the subset of $\mathscr{S}$ consisting of those functions $f \in \mathscr{S}$ for which $f(\mathscr{U})$ is starlike with respect to 0. It is well known that if $f \in \mathscr{S}^*$, then

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > 0,$$

for all $z \in \mathscr{U}$. Aouf and Nasr [1] introduced the class $\mathscr{S}^*(b)$ of starlike functions of order b, where b is a nonzero complex number, as follows:

$$\operatorname{Re}\left\{1 + \frac{1}{b}\left(\frac{zf'(z)}{f(z)} - 1\right)\right\} > 0, \quad z \in \mathscr{U}.$$

Let $\mathscr{S}_c$ denote the set of those functions $f \in \mathscr{S}$ for which there exists a function $g \in \mathscr{S}^*$ such that

$$\operatorname{Re}\left\{\frac{zf'(z)}{g(z)}\right\} > 0,$$

Mathematics subject classification (2010): 30C45.
Keywords and phrases: Close-to-convex functions, logarithmic coefficients, starlike functions.

for all $z \in \mathscr{U}$. The elements of $\mathscr{S}_c$ are called close-to-convex functions. Clearly, $\mathscr{S}^* \subset \mathscr{S}_c$. Al-Amiri and Fernando [2] introduced the class $\mathscr{S}_c(b)$ of close-to-convex functions of complex order b as follows:

$$\operatorname{Re}\left\{1+\frac{1}{b}\left(\frac{zf'(z)}{g(z)}-1\right)\right\}>0,\ \ z\in\mathscr{U}, \tag{1.2}$$

for some starlike function g.

Associated with each $f(z)$ in $\mathscr{S}$ is a well defined logarithmic function

$$\log\frac{f(z)}{z}=2\sum_{n=1}^{\infty}\gamma_n z^n,\ \ z\in\mathscr{U}. \tag{1.3}$$

The numbers γ_n are called the logarithmic coefficients of $f(z)$. Thus the Koebe function $k(z)=z(1-z)^{-2}$ has logarithmic coefficients $\gamma_n=\frac{1}{n}$. It is clear that $|\gamma_1|\leqslant 1$ for each $f(z)\in\mathscr{S}$. The problem of the best upper bounds for $|\gamma_n|$ is still open. In fact even the proper order of magnitude is still not known. It is known, however, for the starlike functions that the best bound is $|\gamma_n|\leqslant\frac{1}{n}$ and that this is not true in general [7, p. 151]; [6, p. 898]; [3, p. 140] and [8].

In the paper [4] it is pointed out that the inequality $|\gamma_n|\leqslant An^{-1}\log n$ (A is an absolute constant) which holds for circularly symmetric functions.

In a recent paper [9], it is presented that the inequality $|\gamma_n|\leqslant\frac{1}{n}$ holds also for close-to-convex functions. However, it is pointed out in [12] that there are some errors in the proof and, hence, the result is not substantiated. It is proved in [10] that there exists a function $f(z)\in\mathscr{S}_c$ such that $|\gamma_n|>\frac{1}{n}$. Furthermore, it is proved in [14] that the inequality $|\gamma_n|\leqslant An^{-1}\log n$ holds for close-to-convex functions, where A is an absolute constant.

In the present paper, we study the logarithmic coefficients of the class $\mathscr{S}_c(b)$.

2. Main results

First, we give the following lemmas.

LEMMA 2.1. [14] *Let $f(z)\in\mathscr{S}$. Then, for $z=re^{i\theta}$, $\frac{1}{2}\leqslant r<1$,*

$$\frac{1}{2\pi}\int_0^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta\leqslant 1+\frac{4}{1-r}\log\frac{1}{1-\sqrt{r}}, \tag{2.1}$$

and

$$\frac{1}{2\pi}\int_{\frac{1}{2}}^{r}\int_0^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta dr\leqslant 1+2\log\frac{1}{1-r}. \tag{2.2}$$

LEMMA 2.2. [5] *Let $f(z)\in\mathscr{S}$, $\tau\in\mathbb{C}$. Then, $z=re^{i\theta}$, $0<r<1$,*

$$\frac{\partial}{\partial\theta}\left(\arg\left(\frac{f(z)}{z}\right)^{\tau}\right)=\tau\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right). \tag{2.3}$$

Proof. It is clear that

$$\frac{zf'(z)}{f(z)} = \frac{1}{i}\frac{\partial}{\partial\theta}\left(\log\frac{f(z)}{z}\right)+1. \tag{2.4}$$

It follows that

$$\operatorname{Re}\frac{zf'(z)}{f(z)} = \operatorname{Im}\left\{\frac{\partial}{\partial\theta}\left(\log\frac{f(z)}{z}\right)\right\}+1 = \frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right)+1. \tag{2.5}$$

Since

$$\frac{zf'(z)}{f(z)} = \frac{1}{i\tau}\frac{\partial}{\partial\theta}\left(\log\left(\frac{f(z)}{z}\right)^{\tau}\right)+1, \tag{2.6}$$

then

$$\operatorname{Re}\frac{zf'(z)}{f(z)} = \frac{1}{\tau}\operatorname{Im}\left\{\frac{\partial}{\partial\theta}\left(\log\left(\frac{f(z)}{z}\right)^{\tau}\right)\right\}+1 = \frac{1}{\tau}\frac{\partial}{\partial\theta}\left(\log\left(\frac{f(z)}{z}\right)^{\tau}\right)+1. \tag{2.7}$$

From (2.5) and (2.7) we obtain

$$\frac{\partial}{\partial\theta}\left(\arg\left(\frac{f(z)}{z}\right)^{\tau}\right) = \tau\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right). \quad \square$$

LEMMA 2.3. [2] *Let $f(z)\in\mathscr{S}_c(b)$. Then for $|z|=r<1$ and $|2b-1|\leqslant 1$*

$$\frac{1-|2b-1|r}{1+r} \leqslant \left|\frac{zf'(z)}{g(z)}\right| \leqslant \frac{1+|2b-1|r}{1-r}. \tag{2.8}$$

THEOREM 2.1. *Let $f(z)\in\mathscr{S}_c(b)$. Then for $n\geqslant 2$,*

$$|\gamma_n| \leqslant An^{-1}\log n \tag{2.9}$$

where A is an absolute constant, and the exponent -1 is the best possible.

Proof. If $f(z)\in\mathscr{S}_c(b)$, then there exist $g(z)\in\mathscr{S}^*$ such that $\operatorname{Re}\left\{1+\frac{1}{b}\left(\frac{zf'(z)}{g(z)}-1\right)\right\}$ >0, $b\neq 0$, $b\in\mathbb{C}$. Write $h(z)=1+\frac{1}{b}\left(\frac{zf'(z)}{g(z)}-1\right)$, then $\operatorname{Re}h(z)>0$. It is clear that

$$h(z) = 2\operatorname{Re}h(z)-\overline{h(z)}.$$

From (1.3), we obtain

$$\frac{zf'(z)}{f(z)} = 1+z\left(\log\frac{f(z)}{z}\right)' = 1+\sum_{k=1}^{\infty}2k\gamma_k z^k. \tag{2.10}$$

Then, for $z=re^{i\theta}$ $(0<r<1)$ and $n=2,3,\ldots$

$$2n\gamma_n = \frac{1}{2\pi i}\oint\limits_{|z|=r}\frac{zf'(z)}{f(z)}z^{-n-1}dz.$$

Hence, we get

$$\begin{aligned}|2n\gamma_n r^n| &= \frac{1}{2\pi}\left|\int_0^{2\pi}\frac{zf'(z)}{f(z)}e^{-in\theta}d\theta\right| = \frac{1}{2\pi}\left|\int_0^{2\pi}[b(h(z)-1)+1]\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| \\ &\leqslant \frac{|b|}{2\pi}\left|\int_0^{2\pi}2\mathrm{Re}\,h(z)\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| + \frac{|b|}{2\pi}\left|\int_0^{2\pi}\overline{h(z)}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| \\ &\quad + \frac{|b-1|}{2\pi}\left|\int_0^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| \\ &= I_1+I_2+I_3. \end{aligned} \tag{2.11}$$

Now, we estimate two terms I_1 and I_2. Write

$$\frac{zf'(z)}{f(z)} = u(re^{i\theta})+iv(re^{i\theta}) \tag{2.12}$$

a) For I_1 :

$$\begin{aligned} I_1 &\leqslant \frac{|b|}{\pi}\int_0^{2\pi}\mathrm{Re}\,h(z)\left|\frac{g(z)}{f(z)}\right|d\theta \leqslant \frac{|b|}{\pi}\left|\int_0^{2\pi}h(z)\left|\frac{g(z)}{f(z)}\right|d\theta\right| \\ &\leqslant \frac{1}{\pi}\left|\int_0^{2\pi}\frac{zf'(z)}{f(z)}e^{i\arg\frac{f(z)}{g(z)}}d\theta\right| + \frac{|1-b|}{\pi}\left|\int_0^{2\pi}\left|\frac{g(z)}{f(z)}\right|d\theta\right| \\ &\leqslant \frac{1}{\pi}\left|\int_0^{2\pi}u(re^{i\theta})e^{i\arg\frac{f(z)}{g(z)}}d\theta\right| + \frac{1}{\pi}\left|\int_0^{2\pi}v(re^{i\theta})e^{i\arg\frac{f(z)}{g(z)}}d\theta\right| + \frac{|1-b|}{\pi}\left|\int_0^{2\pi}\left|\frac{g(z)}{f(z)}\right|d\theta\right| \\ &= 2(J_1+J_2+J_3). \end{aligned} \tag{2.13}$$

Applying the part of integration, (2.5) and (2.3), we have

$$\begin{aligned} J_1 &\leqslant \frac{1}{2\pi}\left|\int_0^{2\pi}e^{i\arg\frac{f(z)}{g(z)}}d\theta\right| + \frac{1}{2\pi}\left|\int_0^{2\pi}\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right)e^{i\arg\frac{f(z)}{g(z)}}d\theta\right| \\ &\leqslant 1+\frac{1}{2\pi}\left|\int_0^{2\pi}\frac{\partial}{\partial\theta}\left(e^{i\arg\frac{f(z)}{z}}\right)e^{-i\arg\frac{g(z)}{z}}d\theta\right| \\ &= 1+\frac{1}{2\pi}\left|\int_0^{2\pi}e^{i\arg\frac{f(z)}{g(z)}}\frac{\partial}{\partial\theta}\left(\arg\frac{g(z)}{z}\right)d\theta\right| \\ &\leqslant 1+\frac{1}{2\pi}\int_0^{2\pi}\left(\left|\frac{\partial}{\partial\theta}(\arg g(z))\right|+\left|\frac{\partial}{\partial\theta}(z)\right|\right)d\theta \end{aligned} \tag{2.14}$$

Since $g(z) \in \mathscr{S}^*$, we have (see [13])

$$\frac{\partial}{\partial\theta}(\arg g(z)) > 0 \text{ and } \int_0^{2\pi} \frac{\partial}{\partial\theta}(\arg g(z)) = 2\pi. \tag{2.15}$$

By applying (2.15), from (2.14) we get

$$\begin{aligned} J_1 &\leqslant 1 + \frac{1}{2\pi}\left(\int_0^{2\pi} \frac{\partial}{\partial\theta}(\arg g(z))d\theta + \int_0^{2\pi} r d\theta\right) \\ &= 1 + \frac{1}{2\pi}(2\pi + 2\pi r) \\ &\leqslant 3. \end{aligned} \tag{2.16}$$

By the Cauchy-Riemann condition, we obtain, for $0 < r_0 < r < 1$

$$v(re^{i\theta}) - v(r_0 e^{i\theta}) = \int_{r_0}^{r} \frac{\partial v(re^{i\theta})}{\partial r} dr = -\int_{r_0}^{r} \frac{1}{r}\frac{\partial u(re^{i\theta})}{\partial\theta} dr. \tag{2.17}$$

By (2.17), we get

$$\begin{aligned} J_2 &\leqslant \frac{1}{2\pi}\left|\int_0^{2\pi} v(r_0 e^{i\theta}) e^{i\arg\frac{f(z)}{g(z)}} d\theta\right| + \frac{1}{2\pi}\left|\int_0^{2\pi}\int_{r_0}^{r} \frac{1}{r}\frac{\partial u(re^{i\theta})}{\partial\theta} e^{i\arg\frac{f(z)}{g(z)}} dr d\theta\right| \\ &= J_{21} + J_{22}. \end{aligned} \tag{2.18}$$

Taking $r_0 = \frac{1}{2}$, it follows that

$$J_{21} \leqslant \max_{\theta\in[0,2\pi]} \left|v(r_0 e^{i\theta})\right| \leqslant \max_{\theta\in[0,2\pi]} \left|\frac{r_0 f'(r_0 e^{i\theta})}{f(r_0 e^{i\theta})}\right| \leqslant \frac{1+r_0}{1-r_0} = 3. \tag{2.19}$$

By the part of integration, we obtain

$$J_{22} \leqslant \frac{1}{2\pi}\left|\int_{r_0}^{r}\int_0^{2\pi} \frac{1}{r} u(re^{i\theta}) e^{i\arg\frac{f(z)}{g(z)}} \left(\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right) - \frac{\partial}{\partial\theta}\left(\arg\frac{g(z)}{z}\right)\right) d\theta dr\right|. \tag{2.20}$$

By (2.5), we have

$$\begin{aligned} \left|\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right) - \frac{\partial}{\partial\theta}\left(\arg\frac{g(z)}{z}\right)\right| &= \left|\left(\operatorname{Re}\frac{zf'(z)}{f(z)} - 1\right) - \left(\operatorname{Re}\frac{zg'(z)}{g(z)} - 1\right)\right| \\ &\leqslant \left|\frac{zf'(z)}{f(z)}\right| + \left|\frac{zg'(z)}{g(z)}\right|. \end{aligned} \tag{2.21}$$

By Schwarz inequality, Lemma 2.1 and (2.21), from (2.20) we get

$$J_{22} \leqslant \frac{1}{\pi}\int_{r_0}^{r}\int_{0}^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta dr + \left(\frac{1}{\pi}\int_{r_0}^{r}\int_{0}^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta dr . \frac{1}{\pi}\int_{r_0}^{r}\int_{0}^{2\pi}\left|\frac{zg'(z)}{g(z)}\right|^2 d\theta dr\right)^{\frac{1}{2}}$$
$$\leqslant 4\left(1+2\log\frac{1}{1-r}\right). \tag{2.22}$$

By (2.19) and (2.22), from (2.18) we obtain

$$J_2 \leqslant 7+8\log\frac{1}{1-r}. \tag{2.23}$$

By (2.16) and (2.23), from (2.13) we have

$$I_1 \leqslant 20+16\log\frac{1}{1-r}+2J_3. \tag{2.24}$$

b) For I_2 :

$$I_2 \leqslant \frac{1}{2\pi}\left|\int_{0}^{2\pi}\overline{\left(\frac{zf'(z)}{f(z)}\right)}e^{-2i\arg\frac{f(z)}{g(z)}}e^{-in\theta}d\theta\right| + \frac{|1-\overline{b}|}{2\pi}\left|\int_{0}^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right|$$
$$= \frac{1}{2\pi}\left|\int_{0}^{2\pi}\frac{zf'(z)}{f(z)}e^{2i\arg\frac{f(z)}{g(z)}}e^{in\theta}d\theta\right| + \frac{|1-\overline{b}|}{2\pi}\left|\int_{0}^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right|. \tag{2.25}$$

From (2.10) we get

$$\frac{zf'(z)}{f(z)}e^{in\theta} = e^{in\theta}\left(1+\sum_{k=1}^{\infty}2k\gamma_k z^k\right) = e^{in\theta}+\sum_{k=1}^{\infty}2k\gamma_k r^k e^{i(n+k)\theta}$$
$$= \frac{1}{i}\frac{\partial}{\partial\theta}\left(\frac{e^{in\theta}}{n}+\sum_{k=1}^{\infty}\frac{2k\gamma_k r^k e^{i(n+k)\theta}}{n+k}\right) = \frac{1}{i}\frac{\partial}{\partial\theta}F(z). \tag{2.26}$$

By the part of integration, we obtain

$$I_2 \leqslant \frac{1}{\pi}\left|\int_{0}^{2\pi}F(z)e^{2i\arg\frac{f(z)}{g(z)}}\left(\frac{\partial}{\partial\theta}\left(\arg\frac{f(z)}{z}\right)-\frac{\partial}{\partial\theta}\left(\arg\frac{g(z)}{z}\right)\right)d\theta\right|$$
$$+\frac{|1-\overline{b}|}{2\pi}\left|\int_{0}^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right|. \tag{2.27}$$

By (2.5) and Schwartz inequality, it follows from (2.27)

$$\begin{aligned} I_2 &\leqslant 2\left(\frac{1}{2\pi}\int_0^{2\pi}|F(z)|^2\,d\theta\right)^{\frac{1}{2}}\left(\frac{1}{2\pi}\int_0^{2\pi}\left(\left|\frac{zf'(z)}{f(z)}\right|+\left|\frac{zg'(z)}{g(z)}\right|\right)^2 d\theta\right)^{\frac{1}{2}} \\ &\quad +\frac{|1-\overline{b}|}{2\pi}\left|\int_0^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}\,d\theta\right| \\ &= 2(L_1L_2)^{\frac{1}{2}}+L_3. \end{aligned} \tag{2.28}$$

Lebedev proves (see [11]) that if $f(z)\in\mathscr{S}$ then

$$\sum_{k=1}^{\infty} k|\gamma_k|\,r^{2k}\leqslant \log\frac{1}{1-r}. \tag{2.29}$$

By the definition of $F(z)$ in (2.26), we obtain from (2.29)

$$L_1=\frac{1}{n^2}+4\sum_{k=1}^{\infty}\frac{k^2|\gamma_k|^2 r^{2k}}{(n+k)^2}\leqslant\frac{1}{n^2}+\frac{4}{n}\sum_{k=1}^{\infty}k|\gamma_k|^2 r^{2k}\leqslant\frac{1}{n^2}+\frac{4}{n}\log\frac{1}{1-r}. \tag{2.30}$$

By Lemma 2.1, it follows that

$$\begin{aligned} L_2 &\leqslant \frac{1}{2\pi}\int_0^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta+2\left(\frac{1}{2\pi}\int_0^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta\,\frac{1}{2\pi}\int_0^{2\pi}\left|\frac{zg'(z)}{g(z)}\right|^2 d\theta\right)^{\frac{1}{2}} \\ &\quad +\frac{1}{2\pi}\int_0^{2\pi}\left|\frac{zg'(z)}{g(z)}\right|^2 d\theta \\ &\leqslant 4\left(1+\frac{4}{1-r}\log\frac{1}{1-\sqrt{r}}\right). \end{aligned} \tag{2.31}$$

Combining (2.30) and (2.31), from (2.28) we get

$$I_2\leqslant 4\left(\frac{1}{n^2}+\frac{4}{n}\log\frac{1}{1-r}\right)\left(1+\frac{4}{1-r}\log\frac{1}{1-\sqrt{r}}\right)^{\frac{1}{2}}+L_3. \tag{2.32}$$

We obtain from (2.11), (2.24) and (2.32) that

$$\begin{aligned} |2n\gamma_n r^n| &\leqslant 20+16\log\frac{1}{1-r}+4\left(\frac{1}{n^2}+\frac{4}{n}\log\frac{1}{1-r}\right)\left(1+\frac{4}{1-r}\log\frac{1}{1-\sqrt{r}}\right)^{\frac{1}{2}} \\ &\quad +I_3+2J_3+L_3. \end{aligned} \tag{2.33}$$

From Lemma 2.1 and Lemma 2.3, we have

$$\begin{aligned} I_3+2J_3+L_3 &= \frac{|b-1|}{2\pi}\left|\int_0^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| + \frac{2|1-b|}{\pi}\left|\int_0^{2\pi}\left|\frac{g(z)}{f(z)}\right|d\theta\right| \\ &\quad + \frac{|1-\overline{b}|}{2\pi}\left|\int_0^{2\pi}\frac{g(z)}{f(z)}e^{-in\theta}d\theta\right| \\ &\leqslant \frac{3|b-1|}{\pi}\int_0^{2\pi}\left|\frac{g(z)}{f(z)}\right|d\theta = \frac{3|b-1|}{\pi}\left(\int_0^{2\pi}\left|\frac{zf'(z)}{f(z)}\right|^2 d\theta \int_0^{2\pi}\left|\frac{g(z)}{zf'(z)}\right|^2 d\theta\right)^{\frac{1}{2}} \\ &\leqslant 6|b-1|\left(1+\frac{4}{1-r}\log\frac{1}{1-\sqrt{r}}\right)^{\frac{1}{2}}\left(\frac{1+r}{1-|2b-1|r}\right). \end{aligned} \tag{2.34}$$

Let $r=1-\frac{1}{n},\ n\geqslant 2,\ |2b-1|\leqslant 1.$ We obtain from (2.33) and (2.34) that

$$\begin{aligned} |\gamma_n| &\leqslant \frac{1}{2n}\left((1-\frac{1}{n}\right)^{-n}\Bigg\{20+16\log n+(1+8n\log n)^{\frac{1}{2}} \\ &\quad \times\left[4\left(\frac{1}{n^2}+\frac{4}{n}\log n\right)^{\frac{1}{2}}+\frac{6(2n-1)|b-1|}{n-(n-1)|2b-1|}\right]\Bigg\}, \end{aligned} \tag{2.35}$$

$$|\gamma_n|\leqslant An^{-1}\log n,$$

where A is an absolute constant. Thus, we have proved Theorem 2.1. □

REMARK 2.2. If we take $b=1$ in (2.35), we have results of [14].

REFERENCES

[1] M. K. AOUF, AND M. A. NASR, *Starlike functions of complex order b*, J. Natural Sci. Math. **25** (1) (1985), 1–12.
[2] H. S. AL-AMIRI, S. AND FERNANDO, *On close-to-convex functions of complex order*, J. Math. & Math. Sci. Vol. **13**, No. 2 (1990), 321–330.
[3] J. ANDERSON, K. BARTH, AND D. BRANNAN, *Research problems in complex analysis*, Bull. London Math. Soc. **9** (1977), 129–162.
[4] Q. DENG, *On circularly symmetric functions*, Appl. Math. Lett. **23** (2010), 1483–1488.
[5] Q. DENG, *On the logarithmic coefficients of Bazilevic functions*, Appl. Math. Comput. **217** (2011), 5889–5894.
[6] P. DUREN,, *Coefficients of univalent functions*, Bull. Amer. Math. Soc. **83** (5) (1977), 891–911.
[7] P. L. DUREN, *Univalent Functions*, Springer-Verlag, New York, 1983.
[8] P. DUREN, Y. LEUNG,, *Logarithmic coefficients of univalent functions*, J. Analyse Math. **36** (1979), 36–43.
[9] M. M. ELHOSH, *On the logarithmic coefficients of close-to-convex functions*, J. Austral. Math. Soc. (Ser. A) **60** (1996), 1–6.
[10] D. GIRELA, *Logarithmic coefficients of univalent functions*, Annales. Acad. Sci. Fen. Math. **25** (2000), 337–350.

[11] N. A. LEBEDEV,, *Applications of area principle to problems on nonoverlapping domains*, Trudy Mat. Inst. Stekliov, **60** (1961), 211–231 (in Russian).
[12] K. PEARCE, *Review of* [9], Math. Reviews, info review no. 96j: 30013, 1996.
[13] CH. POMMERENKE, *Univalent Functions*, Vandenhoeck and Ruprecht, Göttingen, 1975.
[14] Z. YE, *The logarithmic coefficients of close-to-convex functions*, Bulletin of the Institute of Mathematics Academia Sinica (New Series) **3** (3) (2008), 445–452.
[15] Z. YE, *The coefficients of Bazilevic functions*, Complex Variables and Elliptic Equations, **58** (11) (2013), 1559–1567.

(Received February 19, 2014)

Murat Çağlar
Department of Mathematics
Faculty of Science and Letters
Kafkas University, 36100, Kars, Turkey
e-mail: mcaglar25@gmail.com