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SOME SUFFICIENT CONDITIONS FOR FIXED POINTS OF MULTIVALUED NONEXPANSIVE MAPPINGS IN BANACH SPACES

XI WANG, CHIPING ZHANG AND YUNAN CUI

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Abstract. In this paper, we show some sufficient conditions on a Banach space X concerning the generalized von Neumann-Jordan constant, the coefficient $R(1,X)$ and the coefficient of weak orthogonality, which imply the existence of fixed points for multivalued nonexpansive mappings.

1. Introduction

In 1969, Nadler [14] established the multivalued version of Banach contraction principle. By using Edelstein's method of asymptotic centers, T. C. Lim [13] proved that every multivalued nonexpansive self-mapping $T : E \to K(E)$ has a fixed point where E is a nonempty bounded closed convex subset of a uniformly convex Banach space X. In 1990, W.A. Kirk and S. Massa [12] proved that if a nonempty bounded closed convex subset E of a Banach space X has a property that the asymptotic center in E of each bounded sequence of X is nonempty and compact, then every multivalued nonexpansive self-mapping $T : E \to KC(E)$ has a fixed point.

In 2004, Domínguez and Lorenzo [4] proved that every multivalued nonexpansive mapping $T : E \to KC(E)$ has a fixed point where E is a nonempty bounded closed convex subset of a nearly uniformly convex Banach space X. In 2006, S. Dhompongsa et al. [7, 8] introduced the Domínguez-Lorenzo condition and property (D) which imply the fixed point property for multivalued nonexpansive mappings. In 2007, T. D. Benavides and Gavira [2] had established the fixed point property for multivalued nonexpansive mappings in terms of the modulus of squareness, universal infinite-dimensional modulus, and Opial modulus. A. Kaewkhao [11] has established the fixed point property for multivalued nonexpansive mappings in terms of the James constant, the Jordan-von Neumann constant, weak orthogonality. In 2010, T. D. Benavides and Gavira [3] had given a survey of this subject and presented the main known results and current research directions.

Recently, Yunan Cui et al. [6] introduced a new geometric constant $C_{NJ}^{(p)}(X)$ called generalized von Neumann-Jordan constant. In this paper, we show some sufficient conditions on a Banach space X concerning the generalized von Neumann-Jordan constant, the coefficient $R(1,X)$ and the coefficient of weak orthogonality, which imply the existence of fixed points for multivalued nonexpansive mappings.

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2. Preliminaries

Let X be a Banach space. The following constant of a Banach space

$$C_{NJ}(X) := \sup\left\{\frac{\|x+y\|^2+\|x-y\|^2}{2(\|x\|^2+\|y\|^2)} : x,y \in X, \text{not both zero}\right\}.$$

is called the von Neumann-Jordan constant [5], and is widely studied by many authors [14, 7, 2, 3].

The following coefficient is defined by T. D. Benavides [1] as

$$R(1,X) = \sup\{\liminf_{n\to\infty} \|x_n + x\|\},$$

where the supremum is taken over all $x \in X$ with $\|x\| \leqslant 1$ and all weakly null sequences (x_n) in the unit ball B_X such that

$$D[(x_n)] := \limsup_{n\to\infty}(\limsup_{m\to\infty} \|x_n - x_m\|) \leqslant 1.$$

It is clear that $1 \leqslant R(1,X) \leqslant 2$. Some geometric condition sufficient for normal struture in term of this coefficient have been studied in [9, 15].

The coefficient of weak orthogonality $\mu(X)$, defined by the infmum of the set of real numbers $\lambda > 0$ such that

$$\limsup_{n\to\infty} \|x + x_n\| \leqslant \lambda \limsup_{n\to\infty} \|x - x_n\|$$

for all $x \in X$ and all weakly null sequences (x_n) in X [10].

The generalized von Neumann-Jordan constant $C_{NJ}^{(p)}(X)$ [6], defined by

$$C_{NJ}^{(p)}(X) := \sup\{\frac{\|x+y\|^p+\|x-y\|^p}{2^{p-1}(\|x\|^p+\|y\|^p)} : x,y \in X, (x,y) \neq (0,0)\},$$

where $1 \leqslant p < \infty$.

The parametrized formula of this constant is the following

$$C_{NJ}^{(p)}(X) = \sup\left\{\frac{\|x+ty\|^p+\|x-ty\|^p}{2^{p-1}(1+t^p)} : x,y \in S_X, 0 \leqslant t \leqslant 1\right\},$$

where $1 \leqslant p < \infty$.

It was proved that the generalized von Neumann-Jordan constant satisfies the inequality $C_{NJ}^{(p)}(X) \leqslant 2$, and that Banach space X is uniformly non-square if and only if $C_{NJ}^{(p)}(X) < 2$ [6]. If $C_{NJ}^{(p)}(X) < 1 + \frac{1}{\mu(X)^p}$, then the Banach space X has normal structure [15].

Let C be a nonempty subset of a Banach space X. We shall denote by CB(X) the family of all nonempty closed bounded subsets of X and by $KC(X)$ the family of all nonempty compact convex subsets of X . A multivalued mapping $T : C \to CB(X)$ is said to be nonexpansive if

$$H(Tx,Ty) \leqslant \|x-y\|, x,y \in C$$

where $H(.,.)$ denotes the Hausdorff metric on $CB(X)$ defined by

$$H(A,B) := \max\{\sup_{x\in A}\inf_{y\in B}\|x-y\|, \sup_{y\in B}\inf_{x\in A}\|x-y\|\}, A,B \in CB(X).$$

Let $\{x_n\}$ be a bounded sequence in X. The asymptotic radius $r(C,\{x_n\})$ and the asymptotic center $A(C,\{x_n\})$ of $\{x_n\}$ in C are defined by

$$r(C,\{x_n\}) = \inf\{\limsup_n \|x_n - x\| x \in C\}$$

and

$$A(C,\{x_n\}) = \{x \in C : \limsup_n \|x_n - x\| = r(C,\{x_n\})\},$$

respectively. It is known that $A(C,\{x_n\})$ is a nonempty weakly compact convex set whenever C is. The sequence $\{x_n\}$ is called regular with respect to C if $r(C,\{x_n\}) = r(C,\{x_{n_i}\})$ for all subsequences $\{x_{n_i}\}$ of $\{x_n\}$, and $\{x_n\}$ is called asymptotically uniform with respect to C if $A(C,\{x_n\}) = A(C,\{x_{n_i}\})$ for all subsequences $\{x_{n_i}\}$ of $\{x_n\}$. If D is a bounded subset of X , the Chebyshev radius of D relative to C is defined by

$$r_C(D) = \inf_{x\in C}\sup_{y\in D}\|x-y\|.$$

S. Dhompongsa et al. [8] introduced the property (D) if there exists $\lambda \in [0,1)$ such that for any nonempty weakly compact convex subset C of X , any sequence $\{x_n\} \subset C$ which is regular asymptotically uniform relative to C, and any sequence $\{y_n\} \subset A(C,\{x_n\})$ which is regular asymptotically uniform relative to X we have

$$r(C,\{y_n\}) \leqslant \lambda r(C,\{x_n\}).$$

The Domínguez-Lorenzo condition((DL)-condition,in short) introduced in [7] is defined as follows: if there exists $\lambda \in [0,1)$ such that for every weakly compact convex subset C of X and for every bounded sequence $\{x_n\}$ in C which is regular with respect to C,

$$r_C(A(C,\{x_n\})) \leqslant \lambda r(C,\{x_n\}).$$

It is clear from the definition that property (D) is weaker than the (DL)-condition. The next results shows that property (D) is stronger than weak normal structure and also implies the existence of fixed points for multivalued nonexpansive mappings [8]: Let X be a Banach space satisfying ((DL)-condition) property (D), Then X has weak normal structure; Let C be a nonempty weakly compact convex subset of a Banach space X which satisfies ((DL)-condition) the property (D). Let $T : C \to KC(C)$ be a multivalued nonexpansive mapping, then T has a fixed point.

3. The generalized von Neumann-Jordan constant and the coefficient $R(1,X)$

In this section, we show a sufficient condition concerning the generalized von Neumann-Jordan constant, and the coefficient $R(1,X)$, which implies the existence of fixed points for multivalued nonexpansive mappings.

First recall some basic facts about ultrapowers. Let $\mathscr{F}$ be a filter on $\mathbb{N}$. A sequence $\{x_n\}$ in X converges to x with respect to $\mathscr{F}$, denoted by $\lim_{\mathscr{F}} x_n = x$ if for each neighborhood U of x, $\{n \in \mathbb{N}\} \in \mathscr{F}$. A filter U on $\mathbb{N}$ is called to be an ultrafilter if it is maximal with respect to set inclusion. An ultrafilter is called trivial if it is of the form $A : A \in \mathbb{N},\ n_0 \in A$ for some fixed $n_0 \in \mathbb{N}$, otherwise, it is called nontrivial. let $l_\infty(X)$ denotes that the subspace of the product space $\Pi_{n\in\mathbb{N}} X$ equipped with the norm $\|(x_n)\| := \sup_{n\in\mathbb{N}} \|x_n\| < \infty$. Let $\mathscr{U}$ be an ultrafilter on $\mathbb{N}$ and let

$$N_{\mathscr{U}} = \{(x_n) \in l_\infty(X) : \lim_{\mathscr{U}} \|x_n\| = 0\}.$$

The ultrapower of X, denoted by $\tilde{X}$, is the quotient space $l_\infty(X)/N_{\mathscr{U}}$ equipped with the quotient norm, and $(x_n)_{\mathscr{U}}$ denotes the elements of the ultrapower. Note that if $\mathscr{U}$ is non-trivial, then X can be embedded into $\tilde{X}$ isometrically.

THEOREM 1. (Main) *Let C be a weakly compact convex subset of a Banach space X and $\{x_n\}$ is a bounded sequence in C regular with regular to C, then we obtain*

$$r_C(A(C,\{x_n\})) \leqslant \frac{2^{\frac{p-1}{p}} R(1,X)(C_{NJ}^{(p)}(X))^{\frac{1}{p}}}{R(1,X)+1} r(C,\{x_n\}).$$

Proof. Denote $r(C,\{x_n\})$ as r and $A(C,\{x_n\})$ as A. We should assume that $r > 0$, by passing to a subsequence if necessary, we can also assume that $\{x_n\}$ is weakly convergent to a point $x \in C$ and $d = \lim_{n\neq m} \|x_n - x_m\|$ exists. Since $\{x_n\}$ is regular with respect to C, passing through a subsequence does not have any effect to the asymptotic radius of the whole sequence $\{x_n\}$. Observe that the norm is weakly lower semicontinuous, we have

$$\liminf_n \|x_n - x\| \leqslant \liminf_n \liminf_m \|x_n - x_m\| = \lim_{n\neq m} \|x_n - x_m\| = d.$$

Let $\varepsilon > 0$, taking a subsequence if necessary, we can assume that $\|x_n - x\| < d + \varepsilon$ for all n. Let $z \in A$, then we have $\limsup_n \|x_n - z\| = r$ and $\|x - z\| \leqslant \liminf_n \|x_n - z\| \leqslant r$. Denote $R = R(1,X)$, then by definition we have

$$R \geqslant \liminf_n \left\| \frac{x_n - x}{d+\varepsilon} + \frac{z - x}{r} \right\| = \liminf_n \left\| \frac{x_n - x}{d+\varepsilon} - \frac{z - x}{r} \right\|.$$

By the convexity of C, we have $\frac{R-1}{R+1}x + \frac{2}{R+1}z \in C$, since the norm is weak lower semicontinuity, we get

$$\begin{aligned}
&\liminf_n Rr \left\| \frac{x_n - z}{r} + \frac{1}{R}\left(\frac{x_n - x}{d+\varepsilon} - \frac{x - z}{r} \right) \right\| \\
&= \liminf_n Rr \left\| \left(\frac{1}{r} + \frac{1}{R(d+\varepsilon)} \right)(x_n - x) + \left(\frac{1}{r} - \frac{1}{Rr} \right)x - \left(\frac{1}{r} - \frac{1}{Rr} \right) \right\| \\
&\geqslant \|(R-1)x + 2z - (R+1)z\| \\
&= (R+1)\left\| \frac{R-1}{R+1}x + \frac{2}{R+1}z - z \right\| \\
&\geqslant (R+1) r_C(A),
\end{aligned}$$

and

$$\begin{aligned}\liminf_n \left\| R(x_n - z) - \left(\frac{r(x_n - x)}{d+\varepsilon} - (x-z)\right)\right\| \\ \geqslant \left\| \left(R - \frac{r}{d+\varepsilon}\right)(x_n - x) + (R+1)(x-z)\right\| \\ \geqslant | R+1 | r_C(A).\end{aligned}$$

For every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

1. $\|x_N - z\| \leqslant r + \varepsilon$;
2. $\|\frac{r(x_N - x)}{d+\varepsilon} - (x-z)\| \leqslant R(r+\varepsilon)$;
3. $\|R(x_N - z) + \frac{r(x_N-x)}{d+\varepsilon} - (x-z)\| \geqslant \|R+1\| r_C(A)(\frac{r-\varepsilon}{r})$;
4. $\|R(x_N - z) - (\frac{r(x_N-x)}{d+\varepsilon} - (x-z))\| \geqslant | R+1 | r_C(A)(\frac{r-\varepsilon}{r})$.

Now, let $\tilde{u} = R(x_N - z)_{\mathscr{U}}$, $\tilde{v} = (\frac{r(x_N-x)}{d+\varepsilon} - (x-z))_{\mathscr{U}}$. Using the above estimates, we obtain $\|\tilde{u}\| \leqslant R(r+\varepsilon)$, $\|\tilde{v}\| \leqslant R(r+\varepsilon)$ and

$$\begin{aligned}\|\tilde{u} + \tilde{v}\| &= \left\| R(x_N - z) + \frac{r(x_N - x)}{d+\varepsilon} - (x-z)\right\| \\ &\geqslant \|R+1\| r_C(A)\left(\frac{r-\varepsilon}{r}\right),\end{aligned}$$

$$\begin{aligned}\|\tilde{u} - \tilde{v}\| &= \left\| R(x_N - z) - \frac{r(x_N - x)}{d+\varepsilon} - (x-z)\right\| \\ &\geqslant | R+1 | r_C(A)\left(\frac{r-\varepsilon}{r}\right).\end{aligned}$$

By the definition of $C_{NJ}^{(p)}(\tilde{X})$, then

$$\begin{aligned}C_{NJ}^{(p)}(\tilde{X}) &\geqslant \left\{\frac{\|\tilde{u}+\tilde{v}\|^p + \|\tilde{u}-\tilde{v}\|^p}{2^{p-1}(\|\tilde{u}\|^p + \|\tilde{v}\|^p)}\right\} \\ &\geqslant \frac{(R+1)^p}{2^{p-1}R^p}\frac{r_C^p(A)}{r^p}\left(\frac{r-\varepsilon}{r+\varepsilon}\right)^p.\end{aligned}$$

Since the above inequality is true for every $\varepsilon > 0$ and $C_{NJ}^{(p)}(X) = C_{NJ}^{(p)}(\tilde{X})$, we obtain

$$r_C(A(C,\{x_n\})) \leqslant \frac{2^{\frac{p-1}{p}} R(1,X)(C_{NJ}^{(p)}(X))^{\frac{1}{p}}}{R(1,X)+1} r(C,\{x_n\}). \quad \square$$

COROLLARY 1. *Let C be a nonempty bounded closed convex subset of a Banach space X such that $C_{NJ}^{(p)}(X) < \frac{(R+1)^p}{2^{p-1}R^p}$ and $T : C \to KC(C)$ be a multivalued nonexpansive mapping, then T has a fixed point.*

Proof. If $C_{NJ}^{(p)}(X) < \frac{(R+1)^p}{2^{p-1}R^p}$, then X satisfy the (DL)-condition by Theorem 1, so T has a fixed point. □

COROLLARY 2. *Let X be a Banach space such that $C_{NJ}^{(p)}(X) < \frac{(R+1)^p}{2^{p-1}R^p}$, then X has normal structure.*

Proof. By Theorem 1, it is easy to prove that X has weak normal structure. Since $1 \leqslant R(1,X) \leqslant 2$, we obtain $C_{NJ}^{(p)}(X) < \frac{(R+1)^p}{2^{p-1}R^p} < 2$. This implies that X is uniformly nonsquare, then X is reflexive, therefore weak normal structure coincide with normal structure. □

4. The generalized von Neumann-Jordan constant and the coefficient of weak orthogonality

In this section, we show a sufficient condition concerning the generalized von Neumann-Jordan constant, and the coefficient of weak orthogonality, which implies the existence of fixed points for multivalued nonexpansive mappings.

THEOREM 2. *Let C be a weakly compact convex subset of a Banach space X and $\{x_n\}$ is a bounded sequence in C regular with regular to C, then we obtain*

$$r_C(A(C,\{x_n\})) \leqslant \frac{2^{\frac{p-2}{p}}(C_{NJ}^{(p)}(X)(\mu^{2p}+\mu^p))^{\frac{1}{p}}}{\mu^2+1} r(C,\{x_n\}).$$

Proof. Denote $r(C,\{x_n\})$ as r, $A(C,\{x_n\})$ as A and $\mu(X)$ as μ. We should assume that $r > 0$, by passing to a subsequence if necessary, we can also assume that $\{x_n\}$ is weakly convergent to a point $x \in C$ and $z \in A$. Thus,

$$\limsup_n \|x_n - z\| = r$$

$$\limsup_n \|x_n - 2x + z\| \leqslant \mu r.$$

Since $(2/(\mu^2+1))x + (\mu^2-1)/(\mu^2+1)z \in C$ and by the definition of r, we obtain

$$\limsup_n \left\| x_n - \left(\frac{2}{\mu^2+1}x + \frac{\mu^2-1}{\mu^2+1}z \right) \right\| \geqslant r.$$

On the other hand, by the weak lower semicontinuity of the norm, we get

$$\liminf_n \|(\mu^2-1)(x_n-x) - (\mu^2+1)(z-x)\| \geqslant (\mu^2+1)\|z-x\|.$$

For every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that

1. $\|x_N - z\| \leqslant r + \varepsilon$;

2. $\|x_N - 2x + z\| \leqslant \mu(r+\varepsilon)$;

3. $\|x_N - (\frac{2}{\mu^2+1})x + (\frac{\mu^2-1}{\mu^2+1})z\| \geqslant r-\varepsilon$;

4. $\|(\mu^2-1)(x_N-x)-(\mu^2+1)(z-x)\| \geqslant (\mu^2+1)\|z-x\|(\frac{r-\varepsilon}{r})$.

Now, let $u = \mu^2(x_N - z)$ and $v = (x_N - 2x + z)$, then we use the above estimates to obtain $\|u\| \leqslant \mu^2(r+\varepsilon)$ and $\|v\| \leqslant \mu(r+\varepsilon)$, so that

$$\begin{aligned}\|u+v\| &= \|\mu^2((x_N-x)-(z-x))+(x_N-x)+(z-x)\|\\ &= (\mu^2+1)\Big\|(x_N-x)-\frac{\mu^2-1}{\mu^2+1}(z-x)\Big\|\\ &\geqslant (\mu^2+1)\Big\|x_N-\Big(\frac{2}{\mu^2+1}x+\frac{\mu^2-1}{\mu^2+1}z\Big)\Big\|\\ &\geqslant (\mu^2+1)(r-\varepsilon),\end{aligned}$$

$$\begin{aligned}\|u-v\| &= \|\mu^2((x_N-x)-(z-x))-(x_N-x)-(z-x)\|\\ &= \|(\mu^2-1)(x_N-x)-(\mu^2+1)(z-x)\|\\ &\geqslant (\mu^2+1)\|z-x\|\Big(\frac{r-\varepsilon}{r}\Big).\end{aligned}$$

By the definition of $C_{NJ}^{(p)}(X)$ we get

$$C_{NJ}^{(p)}(X) \geqslant \left\{\frac{\|u+v\|^p+\|u-v\|^p}{2^{p-1}(\|u\|^p+\|v\|^p)}\right\} \geqslant \Big(\frac{r-\varepsilon}{r+\varepsilon}\Big)^p \frac{2(\mu^2+1)^p+(\|z-x\|/r)^p}{2^{p-1}(\mu^{2p+\mu^p})}.$$

Let $\varepsilon \to 0^+$, we obtain

$$\|z-x\| \leqslant \frac{2^{\frac{p-2}{p}}(C_{NJ}^{(p)}(X)(\mu^{2p})+\mu^p)^{\frac{1}{p}}}{\mu^2+1}r.$$

Since this inequality holds for arbitrary $z \in A$, we obtain that

$$r_C(A) \leqslant \frac{2^{\frac{p-2}{p}}(C_{NJ}^{(p)}(X)(\mu^{2p}+\mu^p))^{\frac{1}{p}}}{\mu^2+1}r. \quad \square$$

COROLLARY 3. *Let C be a nonempty bounded closed convex subset of a Banach space X such that $C_{NJ}^{(p)}(X) < (\mu^2+1)^p/2^{p-2}(\mu^{2p}+\mu^p)$ and let $T: C \to KC(C)$ be a multivalued nonexpansive mapping. Then T has a fixed point.*

Proof. If $C_{NJ}^{(p)}(X) < (\mu^2+1)^p/2^{p-2}(\mu^{2p}+\mu^p)$, then by Theorem 2, X satisfies the (DL)-condition, then T has a fixed point. $\square$

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Xi Wang
Department of Mathematics
Harbin Institute of Technology
Harbin, 150001, China
e-mail: wangxipuremath@gmail.com

Chiping Zhang
Department of Mathematics
Harbin Institute of Technology
Harbin, 150001, China
e-mail: zcp@hit.edu.cn

Yunan Cui
Department of Mathematics
Harbin University of Science and Technology
Harbin, 150080, China
e-mail: cuiya@hrbust.edu.cn