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UNIFORM ASYMPTOTICS FOR THE TAIL OF THE DISCOUNTED AGGREGATE CLAIMS WITH UTAI CLAIM SIZES

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Abstract. This paper considers a risk model, where the price process of the investment portfolio is described by a geometric Lévy process. When the claim sizes are UTAI, the paper obtains the uniform asymptotics of the tail probability of the discounted aggregate claims and the finite-time ruin probability for the claim sizes with dominated varying distributions. The obtained results extend some existed results.

1. Introduction

In this paper, we consider a risk model, where the claim sizes $\{X_n, n \geqslant 1\}$ are a sequence of nonnegative and identically distributed, but not independent random variables (r.v.s) with common distribution F. The inter-arrival times $\{\theta_n, n \geqslant 1\}$ constitute another sequence of independent and identically distributed (i.i.d) nonnegative r.v.s. The claim arrival times $\tau_n = \sum_{k=1}^{n} \theta_k$, $n \geqslant 1$ and $\tau_0 = 0$ constitute a renewal counting process

$$N(t) = \sup\{n \geqslant 0 : \tau_n \leqslant t\}, \quad t \geqslant 0,$$

which represents the number of claims up to time t and it has a finite mean function $\lambda(t) = E[N(t)] \to \infty$ as $t \to \infty$. We assume that the price process of the investment portfolio is a geometric Lévy process $\{e^{R_t}, t \geqslant 0\}$ with Lévy process $\{R_t, t \geqslant 0\}$, which begins with zero and owns independent and stationary increments. This assumption about the price process has been extensively applied in mathematical finance. One can see [8]–[19].

Suppose that $\{X_n, n \geqslant 1\}$, $\{\theta_n, n \geqslant 1\}$ and $\{R_t, t \geqslant 0\}$ are mutually independent. We use

$$D(t) = \sum_{k=1}^{\infty} X_k e^{-R_{\tau_k}} \mathbf{1}_{\{\tau_k \leqslant t\}} \tag{1.1}$$

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to present the discounted aggregate claims up to time $t \geqslant 0$, in which the indicator function of event E is denoted by $\mathbf{1}_E$. The discounted value of the surplus process with stochastic return on investments of an insurance company is described as

$$U(t) = x + \int_{0-}^{t} c(s)e^{-R_s}ds - D(t), \text{ for any } t \geqslant 0,$$

where $x \geqslant 0$ denotes the initial risk reserve of the insurance company and $c(t)$ is the density function of premium income at time t. Assume that the premium density function $c(t)$ is bounded, i.e. there exists some positive constant H such that $0 \leqslant c(t) \leqslant H$ for all $t \geqslant 0$. For this renewal risk model, the finite-time ruin probability up to time $t \geqslant 0$ can be defined as

$$\psi(x,t) = P\left(\inf_{s\in[0,t]} U(s) < 0 \middle| U(0) = x\right).$$

In this paper, we consider the asymptotics for the tail probability of the discounted aggregate claims, which hold uniformly for each t, such that $\lambda(t)$ is positive. For this, as in [14], define $\Lambda = \{t : 0 < \lambda(t) \leqslant \infty\} = \{t : P(\theta_1 \leqslant t) > 0\}$. If let $\underline{t} = \inf\{t : \lambda(t) > 0\} = \inf\{t : P(\theta_1 \leqslant t) > 0\}$ then

$$\Lambda = \begin{cases} (\underline{t}, \infty] & \text{if} \quad P(\theta_1 = \underline{t}) = 0, \\ [\underline{t}, \infty] & \text{if} \quad P(\theta_1 = \underline{t}) > 0. \end{cases}$$

In order to simplify the investigation, we assume that $\underline{t} = 0$. For any $T \in \Lambda$, set $\Lambda^T = [0, T]$.

In this paper, all limit relationships hold as x tends to ∞, unless noted otherwise. For two positive functions $m(x)$ and $n(x)$, we denote $m(x) \lesssim n(x)$ or $n(x) \gtrsim m(x)$ if $\limsup m(x)/n(x) \leqslant 1$; if $\lim m(x)/n(x) = 1$, then write $m(x) \sim n(x)$; if $\lim m(x)/n(x) = 0$ then write $m(x) = o(n(x))$. For a distribution V on $(-\infty, \infty)$, let $\overline{V}(x) = 1 - V(x)$ be its tail.

This paper mainly discusses the upper tail asymptotic independent claim sizes. A sequence of $\{\xi_n, n \geqslant 1\}$ is called to be upper tail asymptotic independent (UTAI) if for any $x \in (-\infty, \infty)$ and $n \geqslant 1$, $P(\xi_n > x) > 0$, and it holds for any $i \neq j \geqslant 1$ that

$$\lim_{\min\{x,y\}\to\infty} P(\xi_i > x | \xi_j > y) = 0$$

(see [10]).

In the following, we introduce some subclasses of heavy-tailed distributions. A distribution V on $(-\infty, \infty)$ is called to be heavy-tailed distribution, if for any $\lambda > 0$, $\int_{-\infty}^{\infty} e^{\lambda y} V(dy) = \infty$. A distribution V is said to belong to the dominated varying distribution class, which is denoted by $V \in \mathscr{D}$, if for any $0 < y < 1$,

$$\limsup \overline{V}(xy)/\overline{V}(x) < \infty.$$

A distribution V on $(-\infty, \infty)$ is said to belong to the long-tailed distribution class, which is denoted by $V \in \mathscr{L}$, if for any $y \in (-\infty, \infty)$,

$$\lim \overline{V}(x+y)/\overline{V}(x) = 1.$$

For a distribution V on $(-\infty,\infty)$, we denote its upper Matuszewska index by

$$J_V^+ = -\lim_{y\to\infty}\frac{\log \overline{V_*}(y)}{\log y} \quad \text{with} \quad \overline{V_*}(y) := \liminf_{x\to\infty}\frac{\overline{V}(xy)}{\overline{V}(x)} \quad \text{for} \quad y>1,$$

and $L_V = \lim_{y\downarrow 1}\overline{V_*}(y)$. By these definitions, we know that $V\in\mathscr{D} \Leftrightarrow \overline{V_*}(y)>0$ for some $y>1 \Leftrightarrow J_V^+<\infty$ (see [1]).

Reviewing the history of research in the discounted aggregate claims, when $R_t = rt$ for some $r\geqslant 0$ and all $t\geqslant 0$, there are many researchers investigating ruin probabilities, such as [2], [6], [11]–[14], [17], [18], [20], [21] and so on.

When $\{R_t, t\geqslant 0\}$ is a Lévy process, [15] studied the risk model where the claim sizes and the inter-arrival times are two sequences of i.i.d r.v.s and they are mutually independent. [9] considered a dependent risk model, where the claim sizes and the inter-arrival times are also two sequences of i.i.d r.v.s, but there exists a dependence structure between the claim sizes and the inter-arrival times. [19] still considered the case that the claim sizes and inter-arrival times are independent. When the claim sizes are UTAI r.v.s with common distribution belonging to the class $\mathscr{L}\cap\mathscr{D}$, [19] gave the uniform asymptotics of the tail probability of the discounted aggregate claims.

In this paper, we will still investigate the risk model, where the claim sizes and the inter-arrival times are independent. We mainly consider the UTAI claim sizes and extend the result of [19] from the distribution of the claim sizes $F\in\mathscr{L}\cap\mathscr{D}$ to $F\in\mathscr{D}$. This will extend the scope of the applications of the main result.

This paper will suppose that the Lévy process $\{R_t, t\geqslant 0\}$ is right continuous with left limits. Let $E[R_1]>0$, then R_t drifts to ∞ almost surely as $t\to\infty$. We define the Laplace exponent for the Lévy process $\{R_t, t\geqslant 0\}$ as

$$\phi(z) = \log E[e^{-zR_1}], \quad z\in(-\infty,\infty).$$

If $\phi(z)$ is finite then for any $t\geqslant 0$,

$$E[e^{-zR_t}] = e^{t\phi(z)} < \infty$$

(see, e.g. Proposition 3.14 of [3]).

Now we present the main result of this paper.

THEOREM 1.1. *For the discounted aggregate claims* (1.1)*, suppose that the claim sizes* $\{X_n, n\geqslant 1\}$ *are UTAI r.v.s with common distribution* $F\in\mathscr{D}$. *If* $R_t\geqslant 0$ *almost surely for any* $t\geqslant 0$ *then, for each fixed* $T>0$

$$\int_{0-}^{t} P(X_1e^{-R_s}>x)\lambda(ds) \precsim P\big(D(t)>x\big) \precsim L_F^{-1}\int_{0-}^{t} P(X_1e^{-R_s}>x)\lambda(ds)$$

holds uniformly for all $t\in\Lambda^T$.

COROLLARY 1.1. *Under the conditions of Theorem 1.1, if $F \in \mathscr{D}$ then, for each fixed $T > 0$,*

$$L_F \int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds) \lesssim \psi(x,t) \lesssim L_F^{-1} \int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds)$$

holds uniformly for all $t \in \Lambda^T$.

REMARK 1.1. [19] also investigated the discounted aggregate claims (1.1) for heavy-tailed claim sizes. When the distribution of the claim sizes $F \in \mathscr{L} \cap \mathscr{D}$, Theorem 2.1 of [19] obtained the uniform asymptotics of the tail of the discounted aggregate claims. It is well known that $\mathscr{L} \cap \mathscr{D} \subsetneqq \mathscr{D}$, for example the Peter and Paul distribution

$$F(x) = \sum_{k:2^k \leqslant x} 2^{-k}, \quad x \geqslant 0.$$

Then $F \in \mathscr{D}$ but $F \notin \mathscr{L} \cap \mathscr{D}$. For the detailed analysis one can see Goldie [5] and Example 1.4.2 of [4]. Thus Theorem 1.1 extends the scopes of the distributions of claim sizes from the class $\mathscr{L} \cap \mathscr{D}$ to the class $\mathscr{D}$.

2. Proofs of main results

Before giving the proof of Theorem 1.1 and Corollary 1.1, we firstly present some lemmas. The first lemma can be obtained from Proposition 2.2.1 of [1] and Lemma 3.5 of [16].

LEMMA 2.1. *For a distribution V on $(-\infty, \infty)$, if $V \in \mathscr{D}$ then for each $p > J_V^+$,*

(1) there exist positive constants C_1 and D_1 such that the inequality $\frac{\overline{V}(y)}{\overline{V}(x)} \leqslant C_1 \left(\frac{y}{x}\right)^{-p}$ holds for all $x \geqslant y \geqslant D_1$;

(2) $x^{-p} = o\left(\overline{V}(x)\right)$.

The following lemma is attributed to [14].

LEMMA 2.2. *For the renewal counting process $\{N(t), t \geqslant 0\}$, any $v > 0$, and each fixed $T > 0$, it holds that*

$$\lim_{x\to\infty} \sup_{t \in \Lambda^T} \lambda^{-1}(t) E\left[N^v(t) \boldsymbol{1}_{\{N(t) > x\}}\right] = 0.$$

The following lemma can be obtained from Theorem1 of [22].

LEMMA 2.3. *Suppose that $\{\xi_k, k \geqslant 1\}$ are UTAI and nonnegative r.v.s with distributions $V_k \in \mathscr{D}$, $k \geqslant 1$, respectively. The random weights $\{\Theta_k, k \geqslant 1\}$ are a sequence of nonnegative r.v.s and are independent of $\{\xi_k, k \geqslant 1\}$. For some fixed integer $n \geqslant 1$, let $E\Theta_k^p < \infty$, $1 \leqslant k \leqslant n$ for some $p > \max\{J_{V_k}^+, 1 \leqslant k \leqslant n\}$. Then it holds that*

$$\sum_{k=1}^{n} P(\Theta_k \xi_k > x) \lesssim P\left(\sum_{k=1}^{n} \Theta_k \xi_k > x\right) \lesssim L_n^{-1} \sum_{k=1}^{n} P(\Theta_k \xi_k > x),$$

where $L_n = \min\left\{L_{V_k}, 1 \leqslant k \leqslant n\right\}$.

Proof of Theorem 1.1. By (3.1) of the proof of Theorem 2.1 of [19], we get that there exists some positive constant C_2, such that for sufficiently large x and all $t \in \Lambda^T$,

$$\int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds) \geqslant C_2 \overline{F}(x)\lambda(t). \tag{2.2}$$

For each integer $m \geqslant 1$, all $t \in \Lambda^T$ and $x > 0$,

$$\begin{aligned} P(D(t) > x) &= P\left(\sum_{k=1}^{\infty} X_k e^{-R_{\tau_k}} \mathbf{1}_{\{\tau_k \leqslant t\}} > x\right) \\ &= \sum_{n=1}^{\infty} P\left(\sum_{k=1}^{n} X_k e^{-R_{\tau_k}} \mathbf{1}_{\{\tau_k \leqslant t\}} > x, N(t) = n\right) \\ &= \left(\sum_{n=1}^{m} + \sum_{n=m+1}^{\infty}\right) P\left(\sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &=: I_1 + I_2. \end{aligned}$$

For I_2, by Lemma 2.1 and (3.3) of the proof of Theorem 2.1 of [19], for any $p > J_F^+$, it holds uniformly for all $t \in \Lambda^T$ that

$$I_2 \lesssim C_1 \overline{F}(x) E\left[(N(t))^{p+1} \mathbf{1}_{\{N(t) > m\}}\right],$$

which combining with (2.2) and Lemma 2.2 yields that

$$\begin{aligned} &\lim_{m\to\infty} \limsup_{x\to\infty} \sup_{t\in\Lambda^T} \frac{I_2}{\int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds)} \\ &\leqslant \lim_{m\to\infty} \limsup_{x\to\infty} \sup_{t\in\Lambda^T} \frac{I_2}{C_2 \overline{F}(x)\lambda(t)} \\ &\leqslant \frac{C_1}{C_2} \lim_{m\to\infty} \sup_{t\in\Lambda^T} \lambda^{-1}(t) E\left[(N(t))^{p+1} \mathbf{1}_{\{N(t) > m\}}\right] \\ &= 0. \end{aligned} \tag{2.3}$$

Next we estimate I_1. Let $H(y_1, \ldots, y_{n+1})$ be the joint distribution of random vector $(\tau_1, \ldots, \tau_{n+1})$, $n \geqslant 1$. Obviously, for all $1 \leqslant n \leqslant m$, $t \in \Lambda^T$ and $x > 0$,

$$\begin{aligned} &P\left(\sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &= \int_{\{0 \leqslant s_1 \leqslant \ldots \leqslant s_n \leqslant t, s_{n+1} > t\}} P\left(\sum_{k=1}^{n} X_k e^{-R_{s_k}} > x\right) H(ds_1, \ldots, ds_{n+1}). \end{aligned} \tag{2.4}$$

By Lemma 2.3 and (2.3), we get that

$$\begin{aligned} \sum_{k=1}^{n} P\left(X_k e^{-R_{\tau_k}} > x, N(t) = n\right) &\lesssim P\left(\sum_{k=1}^{n} X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &\lesssim L_F^{-1} \sum_{k=1}^{n} P\left(X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \end{aligned}$$

holds uniformly for all $1 \leqslant n \leqslant m$, $t \in \Lambda^T$ and sufficiently large x.

For all $t \in \Lambda^T$ and $x > 0$,

$$\begin{aligned} I_3 &:= \sum_{n=1}^{m} \sum_{k=1}^{n} P\left(X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &= \left(\sum_{n=1}^{\infty} - \sum_{n=m+1}^{\infty}\right) \sum_{k=1}^{n} P\left(X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &=: I_4 - I_5. \end{aligned}$$

Therefore, it holds uniformly for all $t \in \Lambda^T$ and sufficiently large x that

$$I_3 \precsim I_1 \precsim L_F^{-1} I_3. \tag{2.5}$$

For I_4, it holds for all $t \in \Lambda^T$ and $x > 0$ that

$$\begin{aligned} I_4 &= \sum_{k=1}^{\infty} \sum_{n=k}^{\infty} P\left(X_k e^{-R_{\tau_k}} > x, N(t) = n\right) \\ &= \sum_{k=1}^{\infty} P\left(X_k e^{-R_{\tau_k}} > x, N(t) \geqslant k\right) \\ &= \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \end{aligned} \tag{2.6}$$

and

$$\begin{aligned} I_5 &\leqslant \sum_{n=m+1}^{\infty} \sum_{k=1}^{n} P\left(X_k > x\right) P\left(N(t) = n\right) \\ &= \overline{F}(x) \sum_{n=m+1}^{\infty} nP\left(N(t) = n\right) \\ &= \overline{F}(x) E\left[N(t) \mathbf{1}_{\{N(t) > m\}}\right]. \end{aligned} \tag{2.7}$$

By (2.2), (2.7) and Lemma 2.2, we have that

$$\begin{aligned} &\lim_{m\to\infty} \limsup_{x\to\infty} \sup_{t\in\Lambda^T} \frac{I_5}{\int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds)} \\ &\leqslant \lim_{m\to\infty} \limsup_{x\to\infty} \sup_{t\in\Lambda^T} \frac{\overline{F}(x) E\left[N(t)\mathbf{1}_{\{N(t)>m\}}\right]}{\int_{0-}^{t} P(X_1 e^{-R_s} > x)\lambda(ds)} \\ &\leqslant \lim_{m\to\infty} \limsup_{x\to\infty} \sup_{t\in\Lambda^T} \frac{\overline{F}(x) E\left[N(t)\mathbf{1}_{\{N(t)>m\}}\right]}{C_2 \overline{F}(x) \lambda(t)} \\ &= 0. \end{aligned} \tag{2.8}$$

By (2.6) and (2.8), it holds uniformly for all $t \in \Lambda^T$ that

$$I_3 \sim \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds). \tag{2.9}$$

Thus, by (2.5) and (2.9) it holds uniformly for all $t \in \Lambda^T$ that

$$\int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \lesssim I_1 \lesssim L_F^{-1} \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds),$$

which combining with (2.3) gives that

$$\int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \lesssim P(D(t) > x) \lesssim L_F^{-1} \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds)$$

holds uniformly for all $t \in \Lambda^T$. This completes the proof of Theorem 1.1. □

Proof of Corollary 1.1. Next, we follow the line of the proof of Corollary 2.1 of [19] to prove Corollary1.1.

For the upper bound of $\psi(x,t)$, by Theorem 1.1 we know that

$$\psi(x,t) \leqslant P(D(t) > x) \lesssim L_F^{-1} \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \tag{2.10}$$

holds uniformly for all $t \in \Lambda^T$. Then we will deal with the lower bound of $\psi(x,t)$. For any $0 < \varepsilon < 1$ and sufficiently x,

$$\begin{aligned}
\psi(x,t) &= P\left(\inf_{s \in [0,t]} \left\{ D(s) - \int_{0-}^{s} c(h) e^{-R_h} dh \right\} > x \right) \\
&\geqslant P(D(t) > x + HT) \\
&\geqslant P(D(t) > (1+\varepsilon)x) \\
&\gtrsim \int_{0-}^{t} \int_{0}^{1} P(X_1 u > (1+\varepsilon)x) P(e^{-R_s} \in du) \lambda(ds) \\
&= \int_{0-}^{t} \int_{0}^{1} \frac{\overline{F}((1+\varepsilon)x/u)}{\overline{F}(x/u)} \overline{F}(x/u) P(e^{-R_s} \in du) \lambda(ds) \\
&\geqslant \inf_{u \in (0,1]} \frac{\overline{F}((1+\varepsilon)x/u)}{\overline{F}(x/u)} \int_{0-}^{t} \int_{0}^{1} \overline{F}(x/u) P(e^{-R_s} \in du) \lambda(ds) \\
&\gtrsim \overline{F_*}(1+\varepsilon) \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds)
\end{aligned}$$

holds uniformly for all $t \in \Lambda^T$. Note that the facts that the positive Lévy process $\{R_t, t \geqslant 0\}$ has nondecreasing paths and $0 \leqslant c(t) \leqslant H$ are used in the second step, and Theorem 1.1 is used in the fifth step. Let $\varepsilon \to 0$, we have

$$\psi(x,t) \gtrsim L_F \int_{0-}^{t} P\left(X_1 e^{-R_s} > x\right) \lambda(ds) \tag{2.11}$$

holds uniformly for all $t \in \Lambda^T$. Combining (2.10) and (2.11), we finish the proof of Corollary1.1. □

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